PERFORMANCE PAY, TRADE AND INEQUALITY

Germán P. Pupato*
Ryerson University

April 2015

Abstract

This paper introduces moral hazard into a standard general equilibrium model with heterogeneous firms, to study the impact of trade liberalization on wage inequality between homogeneous workers. Trade liberalization operates on two margins of inequality, generating between- and within-firm wage dispersion. While the former channel has been the focus of numerous recent papers, the latter has remained largely overlooked in the literature. In the model, within-firm wage dispersion increases in firm productivity as a result of differential intensity in optimal performance-pay compensation across firms. International trade liberalization triggers labor reallocations towards high productivity firms that result in higher within-firm inequality.

1 Introduction

Our understanding of the impact of international trade on wage inequality has evolved substantially over the last twenty years. In the early 1990’s, most economists dismissed the role of trade as a driving force behind the steep increases in wage inequality that had been observed in many countries around the world since the late 1970’s. Standard factor proportions theory was not easily reconcilable with increasing inequality in developing countries, the absence of significant reallocations of labor across industries and evidence showing that standard human capital variables like education and experience could account for only minor shares of the level and growth of inequality in both developed and developing countries.¹

In recent years, however, a new generation of trade models has caught up to these empirical challenges by shifting its focus from industries to firms, as the basic units of analysis. This research agenda has been fueled by numerous studies documenting a set of stylized facts regarding heterogeneity in firm-level outcomes within industries, including systematic

¹See Katz and Autor (1999) and Goldberg and Pavcnik (2007) for evidence on developed and developing countries, respectively.
differences between exporting and non-exporting firms. Several recent trade theories are motivated by the empirical finding that more productive firms pay higher wages, on average, even after controlling for worker characteristics such as education, experience, occupation and industry affiliation.\(^2\) Using Brazilian data, Helpman et al. (2012) report that 38% of the variance of log wages within sector-occupation cells in 1990 can be accounted for by the variation in wage premia across firms. These facts are compatible with models of firm heterogeneity that feature search frictions and bargaining (Davidson et al. (2008), Helpman et al. (2010), Coçar et al. (forthcoming)), efficiency wages (Verhoogen (2008), Davis and Harrigan (2011)), and fair wage constraints (Egger and Kreickemeier (2009), Amiti and Davis (2011)), in which ex-ante identical workers receive higher wages in more productive firms and wages are systematically related to the export status of the firm.

However, with the exception of Verhoogen (2008) (discussed below), workers employed in the same firm receive identical wages in these models. This literature therefore cannot elucidate an equally sizable component of residual wage inequality (34%) reported in Helpman et al. (2012), namely, within-firm wage dispersion. This evidence is corroborated in recent empirical studies in the United States and several European countries, collected in Lazear and Shaw (2008). Overall, they report that within-firm wage variation ranges from 60% to 80% of the total wage dispersion in each of those countries. In a study of Mexican plants, Frías et al. (2012) find that an exogenous increase in the incentive to export, triggered by the peso devaluation in 1994, resulted in higher within-plant wage dispersion.

The purpose of this paper is to develop a theoretical framework to study this important and largely unexplored dimension of wage inequality, emphasizing its links to international trade. To do so, I extend a standard two-country, general equilibrium model with heterogeneous firms (Melitz (2003)), by adding two key ingredients. First, moral hazard, which generates within-firm wage dispersion between identical workers as firms pay for performance to align the incentives of employees with their best interests. In particular, I study a sequential production process during which workers stochastically make mistakes that are detrimental to product quality. Workers can reduce the frequency of their mistakes by exerting costly effort at each production stage. Firms, in turn, monitor the performance of their workers, observing outcomes (mistakes/successes) but not inputs (effort choices). Importantly, as the frequency of tasks increases, individual performance converges to a Brownian process. This feature of the model allows for a simple characterization of optimal contracts that builds on the seminal work of Holmström and Milgrom (1987).

Second, I introduce cross-firm differences in optimal performance-pay contracts by allowing for complementarity between firm productivity and the performance of workers in generating product quality. Each firm designs a set of contracts, providing incentives to implement desired effort levels. Because high productivity firms have a comparative advantage in generating quality, they find it optimal to offer higher-powered incentives.\(^3\) This implies that, in equilibrium, wages are relatively more dispersed in more productive firms, according to a rich class of inequality measures that includes the variance of log wages and all inequality measures that respect second-order stochastic dominance and scale independence. Importantly, this pattern of inequality across firms is entirely driven by differences in endoge-

---

\(^2\) Evidence of size and exporter wage premia is reported in Bernard and Jensen (1995), Amiti and Davis (2011) and Helpman et al. (2012) for US, Indonesian and Brazilian firms, respectively.

\(^3\) This pattern is consistent with firm-level evidence in Bloom and Van Reenen (2007), who report a positive correlation between the extent to which firms reward performance and total sales in the United States, France, Germany, and the United Kingdom.
nous contracting strategies since, conditional on effort, the stochastic component of worker performance is invariant across firms.

Heterogeneity in performance-pay compensation generates implications for residual wage inequality that remain unexplored in the trade literature. To illustrate these, consider the variance of log wages in any one of the two countries in the model, denoted $Var(w)$. The latter can be decomposed as

$$Var(w) = Var[E(w|\theta)] + E[Var(w|\theta)]$$

where $\theta$ indexes the set of active firms in a given equilibrium. $E(w|\theta)$ and $Var(w|\theta)$ denote the mean and variance of wages across workers employed in firms with productivity $\theta$, respectively. In turn, $Var[\cdot]$ and $E[\cdot]$ integrate over the distribution of workers across firms. The total wage variance is the sum of (i) the variance of mean wages across firms (between-firm inequality) and (ii) the mean of within-firm wage variances (within-firm inequality). As mentioned, recent theoretical studies link trade liberalization to residual wage inequality through mechanisms that operate exclusively on the between-firm component of wage inequality, in which firms of different sizes pay different wages to identical workers but there is no wage dispersion inside firms. The model developed in this paper is, to the best of my knowledge, the first to link trade and residual wage inequality through both channels.

More specifically, the key features and implications of the model regarding the effect of international trade on wage inequality can be summarized as follows:

(a) Performance pay generates wage dispersion within firms. By punishing or rewarding employees according to their performance, high-powered incentives amplify the effect of the idiosyncratic component of performance on wages. This implies $Var(w|\theta) > 0$ in every firm $\theta$.

(b) Different firms design different performance-pay contracts, generating cross-firm variation in the first and second moments of firm-level wage distributions. In particular, more productive firms offer higher-powered incentives and hence $Var(w|\theta)$ increases in $\theta$. Moreover, because equilibrium in the labor market requires workers to be indifferent between employment in any firm, high productivity firms also offer higher expected wages to compensate for higher effort levels. This generates variation in $E(w|\theta)$ across firms, which translates into positive between-firm inequality.

(c) International trade liberalization (i.e., a reduction in bilateral variable trade costs) shapes the distribution of workers across firms through general equilibrium reallocations of labor towards high productivity firms. Specifically, under relatively mild restrictions on the distribution of firm productivity, I show that the distribution of workers across firms in a post-liberalization equilibrium first-order stochastically dominates the corresponding pre-liberalization equilibrium distribution.

In light of (a), (b) and (c), the main result of the paper shows that trade liberalization leads to monotonic increases in within-firm inequality in both countries, as it reallocates workers from low-inequality firms to high-inequality firms. This is valid for symmetric countries and for several decomposable inequality measures, including the Theil index and the

---

4To the extent that data on worker-specific performance histories at the firm level remain hidden to econometricians, wage variation generated by this model should, from an empirical perspective, be understood as residual (i.e. wage variation across workers of identical observable characteristics such as education, experience, occupation, industry affiliation, etc). To the best of my knowledge, this assumption is satisfied in matched employer-employee data sets currently available to study sources of wage variation.
variance of log wages. Moreover, to the extent that trade liberalization triggers firm selection (exit of the least productive firms), the result also holds for countries with asymmetric sizes, trade costs and firm productivity distributions. As in Helpman et al. (2010) and Coçar et al. (forthcoming), however, the effects on between-firm inequality are non-monotonic and difficult to characterize analytically, even in symmetric equilibria.\(^5\)

The mechanism advanced in this paper is both distinct from, and complementary to, the work of Verhoogen (2008). In the latter, an exchange-rate devaluation impacts firm-level wage variances in exporting firms, as the latter upgrade quality by paying relatively higher efficiency wages to (otherwise identical) workers employed in the export production line. However, effort-wage schedules are exogenous and a characterization of equilibrium changes in the distribution of workers across firms is not provided in Verhoogen (2008), preventing a general equilibrium analysis of the impact of trade on within-firm inequality. The latter is the main goal of this paper.\(^6\)

Importantly, the results in this paper do not require the existence of trade-induced effects on firm-level wage distributions. In fact, in the model, there is no quality upgrading or downgrading associated to exporting and hence, given \(\theta\), \(E(w|\theta)\) and \(Var(w|\theta)\) do not change in response to trade liberalization. Heterogeneity in performance-pay contracts ensures that reductions in variable trade costs will still increase within-firm inequality, purely through labor reallocations. Naturally, this mechanism will, in turn, be amplified by increases in the firm-level variances driven by quality upgrading.\(^7\)

There is a class of trade theories in which within-firm wage dispersion is driven by workforce composition, such as Bustos (2011), Monte (2011), Burstein and Vogel (2012), Caliendo and Rossi-Hansberg (2012) and Harrigan and Reshef (forthcoming).\(^8\) In these models, workers are heterogeneous due to differences in skills or human capital, thus they can explain variation in skill premia, as opposed to wage dispersion between identical workers. In addition, wages are determined in competitive labor markets and thus do not contain either firm- or match-specific components. Is it evident, however, that this class of models can potentially generate residual within-firm inequality if augmented with a theory of skills that remain unobservable to the econometrician. Although this argument has not yet been articulated in

\(^{5}\)Under Pareto firm productivity, Helpman et al. (2010) show that between-firm inequality when only some firms export is higher than in both autarky and free trade (see Proposition 3), for all inequality measures that respect second-order stochastic dominance and scale independence. However, no analytical results are provided for changes in variable trade costs between two equilibria in which only some firms export. In Coçar et al. (forthcoming), the effect of increased openness on between-firm inequality is determined by two countervailing forces: increasing wage dispersion across firms and worker reallocations towards high productivity firms. Their model predicts “little if any effect of increased openness” on between-firm inequality.

\(^{6}\)The importance of characterizing equilibrium changes in the distribution of workers across firms in response to trade liberalization cannot be overstated. To illustrate this in a stark way, note that within-firm inequality in an economy could decrease even in a situation in which firm-level wage variances increase in every firm. This would occur if trade liberalization induced labor reallocations towards firms with initially low firm-level wage variances. Observe that the latter are not necessarily low productivity, low effort, firms in Verhoogen (2008), since within-firm wage variances depend not only on the relative wage of high-effort workers but also on their employment shares. For example, for given wages, the firm-level variance of wages is non-monotonic in the share of high-effort workers and will decrease when the latter is sufficiently high.

\(^{7}\)A footnote in page 13 discusses how quality upgrading can be introduced in the model, along the lines of Verhoogen (2008). For evidence linking trade liberalization and quality upgrading, see Verhoogen (2008), Amiti and Khandelwal (2013) and Fan et al. (2014).

\(^{8}\)In Yeaple (2005) and Sampson (2014), differences in workforce composition generate only between-firm wage inequality, since firms hire workers of a single type.
the literature, it is likely to deliver an additional and complementary mechanism to analyze residual wage dispersion within firms.\(^9\) Needless to say, this paper seeks to develop a theory of trade and residual within-firm inequality, which can in principle coexist with alternative theories just as the aforementioned theories of trade and residual between-firm inequality coexist in the literature.

Although a comprehensive empirical assessment of the theory is beyond the scope of this paper, it is worth mentioning several pieces of empirical evidence are consistent with key features of the model. The emphasis on performance pay is motivated by evidence that its prevalence has grown considerably in the last 30 years in the United States. Lemieux et al. (2009) report that, by the late 1990s, performance-pay jobs accounted for as much as 45% of the jobs of male workers and show that this trend can account for a significant share of the growth in wage inequality in the U.S.\(^{10}\) Cross-firm differences in performance-pay policies are, in turn, consistent with evidence from the managerial economics literature. Bloom and Van Reenen (2007) report that large firms tend to rely on incentive pay more intensively than smaller firms. Moreover, the empirical results in Kugler and Verhoogen (2012) support the assumption that larger firms have a comparative advantage in producing high-quality goods. Finally, evidence that trade liberalization induces market share reallocations towards high productivity firms is provided by Pavcnik (2002) and Tre¨pler (2004), for Chile and Canada, respectively.

The outline of the paper is the following. The next section introduces the theoretical framework, sequentially describing the timing of events, market structure, the production technology, the convergence of worker performance to a Brownian process, and consumer preferences. Section 3 studies firms’ optimal performance-pay contracts and profit maximization, embedding the moral hazard problem in a monopolistic competition model with heterogeneous firms. Section 4 analyzes the general equilibrium of the model with two symmetric countries. Section 5 studies how trade liberalization affects the distribution of firm productivity, how labor is reallocated across firms and the implications of the theory for wage inequality between and within firms. The final section discusses extensions and topics for future versions of this paper. The Appendix contains the proofs of the main results.

\(^9\)Note that a trivial extension of this class of models in which skill heterogeneity is assumed fully unobservable to the econometrician is unlikely to provide a useful lens to analyze residual wage dispersion empirically. After all, econometricians typically can predict individual skills, albeit imperfectly, using information routinely available in microdata sets (e.g. education, experience, occupation, industry, etc.), which prevents a direct interpretation of wage variation in these models as purely residual. To make this class of models readily applicable to study the effect of trade liberalization on residual wage dispersion, they would need to be augmented with a theory of what is and what is not observable to the econometrician and, crucially, explain how the latter component varies across firms and responds to general equilibrium shocks such as trade liberalization. A potential disadvantage of this approach is that its applicability would still depend on the quality/detail of the specific data set at hand. On the other hand, for reasons given in footnote 4, wage variation in this paper can be interpreted as purely residual.

\(^{10}\)In particular, using data from the PSID, Lemieux et al. (2009) show that the fraction of U.S. male workers on performance-pay jobs (i.e. workers earning piece rates, commissions, or bonuses) increased from about 30 percent in the late 1970s to over 40 percent in the late 1990s. They also show that wages are less equally distributed on performance-pay than non performance-pay jobs and conclude that the growth of performance-pay has contributed to about 25 percent of the increase in the variance of log wages between the late 1970s and the early 1990s.
2 Model

There are two countries, Home and Foreign. To focus squarely on within-industry, residual wage dispersion, I assume that each country is populated by identical workers that consume a single differentiated good. Countries are identical in terms of market structure, although the size of their labor endowments and the distributions of firm productivity may differ. Variable and fixed trade costs are allowed to be asymmetric for much of the analysis. Firms endogenously choose the level and quality of output, and market(s) to serve in the presence of international trade costs. The main departure from the literature is the existence of moral hazard in the production process. Firms respond by tying compensation to individual performance, generating between- and within-firm wage inequality.

I focus on the description of Home’s economy and use an asterisk to denote Foreign variables.

2.1 Setup

The timing of events in the model combines elements of Holmström and Milgrom (1987) and the static formulation of Melitz (2003). A competitive fringe of risk neutral firms may potentially enter the differentiated sector. Upon incurring a sunk entry cost of $f_e > 0$ units of the differentiated good, a firm observes its productivity $\theta$, independently drawn from a distribution $G_\theta(\theta)$, with positive and bounded support $[\theta_L, \theta_H]$. Firms then decide whether to exit, produce solely for the domestic market, or produce for both the domestic and export markets. A successful entrant becomes a monopolistic producer of a single variety of good $X$. Production requires a fixed cost of $f_d > 0$ units of the domestic differentiated good. In addition, exporting involves a fixed cost of $f_x > 0$ units of the domestic differentiated good and an iceberg variable trade cost, such that $\tau > 1$ units of the firm’s output must be exported for one unit to arrive in the foreign market. Since all firms with the same productivity behave symmetrically in equilibrium, I index firms and varieties by $\theta$ from now onward.

The quantity and quality of firm output depend on both the mass and effort of workers allocated to a sequential production process with stochastic performance. In the presence of moral hazard, each firm hires a mass of workers and designs performance-pay contracts to implement desired effort sequences. Workers can accept or reject contracts prior to starting production. In the former case, each worker chooses effort at each stage of the production process, as a function of its (publicly observed) personal history of realized performance in previous tasks. At the end of the production process, contracts are executed and consumption takes place.

Because workers are homogeneous, equilibrium in the labor market requires that every contract offered by any firm should be individually rational, yielding the same expected utility, denoted $\pi$. The latter is endogenously determined by either a labor market clearing condition in the single-sector model or by an indifference condition if an outside sector is added to the model.

\[\text{\footnotesize\textsuperscript{11}}\text{Contracts yielding a lower expected utility than the outside option would fail to attract workers. Exceeding $\pi$ would not be profit-maximizing.}\]
2.2 Sequential Production with Stochastic Performance

Firm output is vertically and horizontally differentiated. Physical output of each variety \((y)\) increases in firm productivity \((\theta)\) and the mass of workers \((h)\) allocated to the production process; i.e., \(y = y(\theta, h)\). In turn, product quality \((q)\) increases in firm productivity and decreases in the average number of mistakes \((n)\) that workers make during the production process; i.e., \(q = q(\theta, n)\). I will differ imposing additional structure on \(y(\cdot, \cdot)\) and \(q(\cdot, \cdot)\) until Section 3.2, since this is not necessary to characterize optimal contracts. The latter are determined by the performance of workers throughout the production process, as described in the remainder of this section.

In every firm, the production process requires each worker to perform a sequence of \(T\) symmetric tasks, indexed by \(\tau = 1, ..., T\). Each task spans an interval of time of length \(\Delta \equiv 1/T\). Worker \(i\) chooses a sequence \(\{\epsilon_i^\Delta\}_{\tau=1}^T\) of possibly history-dependent effort levels for each task \(\tau\), where \(\epsilon_i^\Delta \in [\epsilon_L, \epsilon_H] \subset \mathbb{R}_{++}\). This choice generates a stochastic sequence of worker-specific performance outcomes \(\{Z_i^\tau\}_{\tau=1}^T\), where \(Z_i^\tau\) equals 1 if worker \(i\) successfully completes task \(\tau\) and equal to \(-1\) in the event of a mistake, for \(\tau = 1, ..., T\). For a fixed \(\Delta\), the probability of success in any task \(\tau\), denoted \(\pi_i^\Delta(\cdot)\), is given by

\[
\pi_i^\Delta(\cdot) = P(Z_i^\Delta = 1 | z_{i1}, ..., z_{i\tau-1}) = \frac{1}{2} + \mu(\epsilon_i^\Delta) \frac{\Delta^{1/2}}{2},
\]

where \(\mu(\cdot)\) is continuous and, since \(\epsilon_i^\Delta \in [\epsilon_L, \epsilon_H]\), bounded.\(^\text{13}\) The expected performance of worker \(i\) in task \(\tau\) is \(E(z_i^\tau | z_{i1}, ..., z_{i\tau-1}) = \mu(\epsilon_i^\Delta) \Delta^{1/2}\) and thus it is also natural to assume that \(\mu(\cdot)\) is increasing. Note that \(z_i^\tau\) is independent of firm productivity and, conditional on effort, independent of \(z_{i\tau'}\) for any two tasks \(\tau\) and \(\tau'\) and any two workers \(i\) and \(i'\) (unless, of course, \(\tau = \tau'\) and \(i = i'\)). The randomness of a task’s outcome captures unmodeled determinants of a worker’s performance such as unobserved skills and idiosyncratic variation in the quality of inputs used in the production process.

Let \(Z_i^\Delta\) denote the normalized cumulative performance of worker \(i\) up to task \(\tau\), \(Z_i^\Delta = 1/2 \sum_{\tau'=1}^\tau z_{i\tau'}\). Equivalently, \(-Z_i^\Delta\) is the normalized number of mistakes in excess of successes of worker \(i\) up to task \(\tau\); i.e., the net number of mistakes. To characterize the convergence of the path of cumulative performance as the duration of tasks \(\Delta\) approaches zero, I embed the discrete process \(\{Z_i^\tau\}_{\tau=1}^T\) in continuous time by linearly interpolating between the points \(0, 0\), \((\Delta, Z_i^\Delta)\), \((2\Delta, Z_i^\Delta)\), ..., \((1, Z_i^\Delta)\). In other words, I construct a function \(Z_i^\Delta(t)\) satisfying

\[
Z_i^\Delta(t) = \left(1 - \frac{t}{\Delta} + \left[\frac{t}{\Delta}\right]\right) Z_i^\Delta([t/\Delta]) + \left(\frac{t}{\Delta} - \left[\frac{t}{\Delta}\right]\right) Z_i^\Delta([t/\Delta]+1),
\]

for \(t \in [0, 1]\) and the initial condition \(Z_i^\Delta(0) = 0\), where \([x]\) is the integer part of \(x \geq 0\). Note that \(Z_i^\Delta(t)\) is a random element of the space of continuous functions, \(C[0, 1]\). Similarly, let \(\epsilon_i^\Delta(t)\) denote a continuous-time representation of \(\{\epsilon_i^\Delta\}_{\tau=1}^T\), where \(\epsilon_i^\Delta(t) \equiv \epsilon_i^\Delta([t/\Delta])\). Endowing \(C[0, 1]\) with the uniform metric, I obtain the following result.

\(^\text{12}\) \(\mu_{\text{min}}\) is the lowest feasible effort level for any worker. For the purpose of describing technology, it suffices to take the sequence of effort levels as given. Optimal effort choices are analyzed in Section 3.1 in the continuous-time limit of this production process.

\(^\text{13}\) For any positive \(\Delta\), \(|\mu(\cdot)| \leq \Delta^{-1/2}\) is necessary and sufficient for \(0 \leq \pi_i^\Delta \leq 1\). When \(\Delta \to 0\), the former condition becomes innocuous. However, boundedness of \(\mu(\cdot)\) is still necessary to establish Lemma 1.
Lemma 1 In the sequential production process over the unit time interval with task duration $\Delta = 1/T$, consider a sequence of effort $\{\epsilon_i(\tau)\}_{\tau=1}^T$ and the corresponding process of cumulative performance $\{Z_{i\tau}^\Delta\}_{\tau=1}^T$ for worker $i$. Suppose that $\epsilon_i(\tau) \to \epsilon_i(\cdot)$ a.s. as $\Delta \to 0$, for $t \in [0,1]$. If $\mu(\cdot)$ is continuous then, as $\Delta \to 0$, $Z_{i\tau}^\Delta(t)$ converges in distribution to a stochastic process $Z_i(t)$, such that:

$$Z_i(t) = \int_0^t \mu(\epsilon_i(t')) dt' + B_i(t),$$

for $t \in [0,1]$, where $B_i(t)$ is a Wiener process on $0 \leq t \leq 1$, such that for all $i$, $B_i(0) = 0$ a.s. and $E[B_i(1)^2] = 1$.

Proof. Appendix

The crux of this result is showing that deviations of cumulative performance $\{Z_{i\tau}^\Delta\}_{\tau=1}^T$ from its expected value follow a martingale process. Convergence to a standard Brownian motion in the space $C[0,1]$ is then an application of martingale limit theory. In particular, the proof of Lemma 1 relies on a result due to Brown (1971). The assumptions of Bernoulli task outcomes and unidimensional effort are not essential.

Lemma 1 states that, when task duration approaches zero, the path of cumulative performance of worker $i$ converges to a Brownian process whose (random) drift is a function of the worker’s effort choices. The remainder of the paper concentrates on this limiting case, due to computational ease of optimal contracts. The value of Lemma 1 is to provide an economically relevant interpretation of this continuous-time environment as the limit of the sequential production process introduced above.

To map outcomes of the continuous-time production process into product quality, recall that $-Z_{i\tau}^\Delta$ is the net number of mistakes of worker $i$ in the production process with task length $\Delta$. Therefore, I define the average number of (net) mistakes as follows:

$$n \equiv h^{-1} \int_0^h -Z_i(1) di.$$

Because the $Z_i$’s are conditionally independent across workers, the strong law of large numbers implies that the firm fully diversifies the impact of idiosyncratic individual performance, $B_i(1)$, on $n$. Moreover, in the next section, I show that firms optimally implement non-stochastic effort sequences. These observations imply that $n$ is almost surely a constant and thus equilibrium firm-level variables such as quality, output price, revenue and profits are deterministic with probability one.

2.3 Demand

Home is populated by a continuum of identical risk-neutral workers of mass $L$. The preferences of any worker $i$ depend on the consumption of a differentiated product $X_i$ and on the sequence of effort $\epsilon_i \equiv \{\epsilon_i(t) ; t \in [0,1]\}$ exerted during the production process:

$$U(X_i, \epsilon_i) = \frac{X_i}{\exp(\int_0^1 k(\epsilon_i(t)) dt)},$$

Hall and Heyde (1980) provide a comprehensive treatment of this literature.

Hellwig and Schmidt (2002) generalize these assumptions in the context of a principal-agent model.
where \( k(\cdot) \) is an increasing and convex instantaneous cost-of-effort function. \( X_i \) indexes the consumption of a continuum of horizontally and vertically differentiated varieties, defined as
\[
X_i = \left[ \int_{j \in J} (q(j)x_i(j))^{\frac{\nu-1}{\nu}} dj \right]^\frac{\nu}{\nu-1},
\]
where \( j \) indexes varieties, \( J \) is the set of varieties available in the market, \( x_i(j) \) and \( q(j) \) denote the consumption and quality of variety \( j \), respectively, and \( \nu > 1 \) is the elasticity of substitution across varieties. The quality-adjusted price index dual to \( X_i \) is denoted by \( P. \)

For a worker earning a wage \( w_i \), the familiar two-stage budgeting solution yields \( PX_i = w_i \) and individual demand \( x_i(j) = w_i q(j)^{\nu-1} p(j)^{-\nu}/P^{1-\nu} \). Other than for final consumption, differentiated products are also demanded by firms to set up production and export activities (fixed costs). These activities are assumed to use the output of each variety in the same way as is demanded by final consumers. Denoting total expenditure on the differentiated good by \( E \), the aggregate demand for variety \( j \), denoted \( x(j) \), can then be written as
\[
x(j) = q(j)^{\nu-1} p(j)^{-\nu} E.
\]
The aggregate expenditure on variety \( j \) in Home equals the revenue of producer \( j \) in Home, denoted \( r(j) \). Therefore,
\[
r(j) = p(j)x(j) = Aq(j)\rho x(j)^\rho,
\]
where \( A \equiv P^\rho E^{1-\rho} \) and \( \rho \equiv (\nu - 1)/\nu. \)

For expository purposes, it is convenient to simplify the notation by setting the aggregate consumption index in Home to be the numeraire \( (P = 1) \). To express the utility of domestic consumers solely as a function of income and effort choices; i.e., \( U(X_i, \epsilon_i) = U(w_i, \epsilon_i). \)

### 3 The Firm’s Problem

This section studies the problem of firm \( \theta \) located in Home, in two steps. The first step takes firm employment and output quality as given, while seeking to characterize the design of optimal contracts to attain the targeted quality at minimum cost. The second step sets up the profit maximization problem, in which the firm determines employment, quality and whether to export given demand in the domestic and foreign markets.

#### 3.1 Optimal Performance-pay Contracts

The cost of attaining a given quality \( q_0 = q(\theta, n_0) \) per unit of output is determined by the cost of providing incentives such that the average net number of mistakes in the production process is \( n_0 \). A *performance-pay contract* for any worker \( i \) is an arbitrary function \( w_i = w_i(Z^1_i) \), stipulating the wage of worker \( i \) based on the realized path of individual performance \( Z^1_i \); i.e., \( Z^1_i \equiv \{ Z_i(t); t \in [0, 1] \}. \)

Workers accept or reject contracts prior to starting production at
\[
\text{for } \int_{j \in J} \left( \frac{p(j)}{q(j)} \right)^{1-\nu} dj. \]

Although potentially relevant to study within-firm wage variation, this paper does not deal with any form of group-based compensation schemes. The emphasis on individual incentives can be motivated empirically.
time $t = 0$, select effort in each task $t$ having observed $\{Z_i(t'); t' \in [0,t]\}$ and receive wages upon completion of all tasks at time $t = 1$.

To attain $g_0$, a firm employing $h$ workers designs a set of contracts and effort sequences $\{w_i, \epsilon_i; i \in [0,h]\}$ that minimize expected total compensation subject to: (i) inducing fewer than $n_0$ mistakes per worker, (ii) the stochastic processes for individual performance, (iii) incentive compatibility constraints and (iv) participation constraints:

$$\min_{\{w_i, \epsilon_i; i \in [0,h]\}} \int_0^h E \left[ w_i(Z_i^1) \right] di \quad (5)$$

s.t.  

(i) $n_0 \geq h^{-1} \int_0^h Z_i(1) di$

(ii) $Z_i(t) = \int_0^t \mu (\epsilon_i (t')) dt' + B_i(t)$, for $i \in [0,h]$

(iii) $\epsilon_i \in \arg \max \ E \left[ U(w_i, \epsilon_i) \right]$, for $i \in [0,h]$

(iv) $E \left[ U(w_i, \epsilon_i) \right] \geq \overline{u}$, for $i \in [0,h]$

Proposition (1) characterizes the solution to this problem for the case in which $n_0 \in N \equiv [-\mu (\epsilon_H), -\mu (\epsilon_L)]$. The infimum of $N$ ensures that $n_0$ is technologically feasible, a necessary condition for the existence of a solution in (5). In turn, the supremum of $N$ makes the moral hazard problem interesting, by requiring the firm to implement effort levels greater than $\epsilon_L$.\(^\dagger\)

To establish the result, I need:

**Assumption 1** For all $\epsilon \in [\epsilon_L, \epsilon_H]$,\n
(a) $\mu(\epsilon)$ is $C^1$, strictly increasing and strictly concave.
(b) $k(\epsilon)$ is $C^1$, strictly increasing and convex.

**Proposition 1 (Cost-minimizing contracts)** Suppose that $n_0 \in N$. Then, under Assumption (1), there exists an a.s. unique global minimizer in problem (5), denoted $\{w_i^*, \epsilon_i^*; i \in [0,h]\}$, such that:

(a) **Effort:** $\epsilon_i^*(t) = \epsilon^*$, for all $t \in [0,1]$ and $i \in [0,h]$, where

$$\epsilon^* = \mu^{-1} (-n_0).$$

(b) **Contract:** $\log (w_i^*) = \alpha + \beta Z_i(1)$, for all $i \in [0,h]$, where

$$\beta = k'(\epsilon^*) / \mu'(\epsilon^*),$$

$$\alpha = \ln \overline{u} + k(\epsilon^*) - \frac{k'(\epsilon^*)}{\mu'(\epsilon^*)} \mu(\epsilon^*) - \frac{1}{2} \left[ \frac{k'(\epsilon^*)}{\mu'(\epsilon^*)} \right]^2.$$

(c) **Performance:** $Z_i(1) \sim N (\mu(\epsilon^*), 1).

\(^\dagger\)Lazear and Shaw (2007) report that the share of large US firms in which more than 20 percent of their workforce is subject to some form of individual incentives, like a performance bonus, has grown from 38 percent in 1987 to 67 percent in 1999. The comparable share of firms using any form of ‘gain-sharing’ or group-based incentives was 7 percent in 1987 and 24 percent in 1999.

\(^\dagger\)If $n_0 \geq -\mu (\epsilon_L)$, the firm can satisfy (i) by simply offering a constant wage that ensures participation, trivializing the moral hazard problem.
Proof. Appendix.

The solution to problem (5) has several important features. First, the optimal contract for worker \( i \) is a log-linear function of \( i \)'s cumulative performance at time \( t = 1 \) and implements a constant effort in each task of the production process. The model thus inherits the simple structure of contracts in Holmström and Milgrom (1987). As in that paper, tasks (time periods) are technologically independent and consumption takes place after production, eliminating any scope for improved statistical inference and for consumption smoothing throughout the production process. A conceptually significant departure relative to Holmström and Milgrom (1987) is the specification of the objective functions of firms and workers. In particular, the firm’s cost minimization problem (5) arises naturally in the context of the broader profit maximization problem studied in the next section. Moreover, the utility function (3) plays a key role in ensuring that wages are positive for all realizations of individual performance \( Z_i(1) \).

Because wages fuel the demand side of the model, this is an essential property for embedding the moral hazard problem in general equilibrium.

Second, the firm’s cost minimizing strategy is to offer identical contracts to its \( h \) employees. In principle, the firm could offer different contracts to different workers, yet this is not cost-effective. The symmetry of optimal effort levels - part (a) - follows from the convexity of the effort cost function \( k(\cdot) \) and the concavity of \( \mu(\cdot) \). Intuitively, the convexity of \( k(\cdot) \) implies that the cost of compensating a worker for a higher-than-average effort exceeds the cost reduction of inducing another worker to exert a lower-than-average effort level. In addition, the concavity of \( \mu(\cdot) \) implies that a higher-than-average effort of some worker does not compensate the mistakes incurred by a lower-than-average effort of another worker. The strict concavity of \( \mu(\cdot) \) ensures the uniqueness of the optimal effort and contract, up to an almost sure equivalence.

Third, it is straightforward to verify that, under Assumption (1), \( \beta \) is increasing in \( \epsilon^* \). Incentive compatibility requires the intensity of performance pay (proxied by \( \beta \) ) to increase in effort, which is consistent with empirical studies documenting performance gains from performance pay. The firm adjusts the fixed component of compensation \( \alpha \) to ensure that the participation constraint is satisfied with equality.

Proposition (1) has implications for the distribution of wages within the firm, which are summarized as follows.

**Corollary 1 (Firm-level wages and inequality)** Suppose that the firm implements a constant effort \( \epsilon \in [\epsilon_L, \epsilon_H] \) such that \( \epsilon_i(t) = \epsilon \) for all \( t \in [0, 1] \) and \( i \in [0, h] \). Then, under Assumption (1):

(a) The firm-level mean wage is

\[
E[w^*_i|\epsilon] = \bar{w}e^{k(\epsilon)}.
\]

(b) Firm-level wage inequality increases in \( \epsilon \), according to:

(i) the variance of log wages,

(ii) all inequality measures that respect second-order stochastic dominance and scale independence.

---

19In Holmström and Milgrom (1987), firms and workers have negative exponential (CARA) objective functions defined over cumulative performance at \( t = 1 \) and compensation, respectively. Moreover, effort costs are measured in monetary units. The optimal contract is a linear function of a normally distributed random variable and thus the support of the wage distribution is \( \mathbb{R} \).

20See, for example, Parent (1999), Lazear (2000) and references cited in Lazear and Shaw (2007).
Proof. Appendix. ■

Part (a) of Corollary (1) and the SLLN imply that the average wage paid by the firm to implement effort \( \epsilon \), denoted \( \omega(\epsilon) \), equals \( E[w_i^*|\epsilon] \) with probability 1. As with output quality, the firm effectively diversifies the impact of idiosyncratic performance on the average wage paid to its employees. In the next section, I take the approximation to be exact and treat average wages as deterministic in the firm’s profit maximization problem. Part (b) builds on the observation that, by Proposition (1), the distribution of firm-level wages is log-normal. It states that the optimal provision of incentives generates higher firm-level inequality in high-effort firms, according to a wide range of scale-independent inequality measures.

The next section endogeneizes the choice of effort. In combination with Corollary (1), this will provide a mapping between firm-level wages and firm productivity that enables a tractable comparison of wage distributions across firms. In particular, after establishing the conditions under which effort increases in firm productivity, Corollary (1) will imply that both average wage and firm-level wage inequality increase in firm productivity. This result will play a key role in the analysis of the impact of international trade on wage inequality.

### 3.2 Profit Maximization

If price and quality discrimination across markets is feasible, then by (4) revenues from domestic and foreign sales are given by \( r_d = Aq^*_d y^*_d \) and \( r_x = A^*q^*_x [y_x/\ell]^\rho \), where \( r_m \), \( q_m \) and \( y_m \) denote revenue, quality and output in market \( m = \{d, x\} \), respectively. I assume that the firm’s output in \( m \) is linear in the mass of workers allocated to that production line, \( h_m \),

\[
y_m = \theta^s h_m, \quad s \geq 0.
\]  

The analysis nests the case of identical labor productivity across firms \( (s = 0) \). Recall that fixed costs are measured in units of the domestic differentiated good, whose price is normalized to one. From these observations, it follows that the profit maximization problem of firm \( \theta \) located in Home is additively separable in domestic and foreign profits and can be written as

\[
\Pi(\theta) \equiv \max_{\epsilon_m \in [\ell_L, \ell_H], \atop q_m \in [q_L, q_H], \atop y_m \geq 0, \atop I_x \in \{0, 1\}} Aq^*_d y^*_d \frac{\omega(\epsilon_d)}{\theta^s} y_d - f_d + I_x \left[ A^*q^*_x [y_x/\ell]^\rho - \frac{\omega(\epsilon_x)}{\theta^s} y_x - f_x \right],
\]  

where \( q_\ell = q(\theta, -\mu(\epsilon)) \), \( \ell = \{m, L, H\} \) and \( m = \{d, x\} \), and \( I_x \) equals 1 if firm \( \theta \) exports and 0 otherwise.\(^{21}\) The average wage function, \( \omega(\cdot) \), is obtained from part (a) of Corollary (1).

Profits are strictly concave in output and marginal revenue of output is infinite as output approaches zero. Therefore, for any market \( m = \{d, x\} \): (i) the first-order condition with respect to \( y_m \) is necessary and sufficient to maximize profits in \( m \), for any given quality \( q_m \in [q_L, q_H] \); (ii) corner solutions for output \( (y_m = 0) \) can be ruled out. I thus solve problem (7) in three steps. First, assuming \( I_x = 1 \), I compute the optimal output in each market for a given quality \( q_m \), denoted, \( y_m(q_m) \). Second, I compute the optimal \( q_m \), accounting for its effect on \( y_m(q_m) \). Finally, I determine whether exporting is profit maximizing.

\(^{21}\) Allowing for quality discrimination, the firm can in principle choose to supply different product qualities in the home and foreign markets. If so, workers allocated to different ‘production lines’ will earn different expected wages. Still, in equilibrium workers are indifferent between employment in either production line because every contract yields the same expected utility.
Let \( c^\theta (q) \equiv \omega(\epsilon(\theta, q))/\theta^\rho \), where \( \epsilon(\theta, q) \) is implicitly defined by \( q = q(\theta, -\mu(\epsilon)) \). Then \( c^\theta (q) \) is the (factory) unit-cost function in firm \( \theta \) when output quality is \( q \). Assume \( I_x = 1 \) and express profits in market \( m = \{d, x\} \) as \( \Pi_m \equiv \eta_m q_m^p y_m^q - c^\theta (q_m) y_m - f_m \), where \( \eta_d \equiv A \) and \( \eta_x \equiv A^* \rho^{-\rho} \).

For any fixed \( q_m \in [q_L, q_H] \), output in market \( m \) maximizes \( \Pi_m \) if and only if it equals the marginal revenue of output and the marginal cost of output,

\[
\eta_m q_m^p [y_m(q_m)]^{\rho-1} = c^\theta (q_m), \quad \text{for } m = \{d, x\}.
\] (8)

Solving for \( y_m(q_m) \) from (8), substituting it in \( \Pi_m \) and rearranging yields,

\[
\Pi_m(q_m) = \left[ (\rho)^{\rho/(1-\rho)} - (\rho)^{1/(1-\rho)} \right] (\eta_m)^{1/(1-\rho)} \left( \frac{q_m}{c^\theta (q_m)} \right)^{\rho/(1-\rho)} - f_m.
\] (9)

Next, \( 0 < \rho < 1 \) implies \( (\rho)^{\rho/(1-\rho)} > (\rho)^{1/(1-\rho)} \). Therefore, quality \( q_m(\theta) \) is profit maximizing for firm \( \theta \) in market \( m \) if and only if \( q_m(\theta) \) minimizes the average cost of quality (per unit of output) in \( m \), \( c^\theta (q) / q \). By Weierstrass theorem, if \( c^\theta \) is continuous then \( q_m(\theta) \) exists, since \( q \in [q_L, q_H] \). If, in addition, \( c^\theta \) is differentiable and \( q_m(\theta) \in (q_L, q_H) \), then

\[
c^\theta (q_m(\theta)) = \frac{c^\theta (q_m(\theta))}{q_m(\theta)}, \quad \text{for } m = \{d, x\}.
\] (10)

Geometrically, the marginal and average costs of quality intersect at \( q_m(\theta) \).

Importantly, the average cost of quality is independent of the variable trade cost \( \tau \). Therefore, trade liberalization does not induce quality upgrading or downgrading at the firm level, in either market.\(^{22}\) Moreover, if \( q_m(\theta) \) is the unique global minimizer of \( c^\theta (q) / q \) then \( q_d(\theta) = q_x(\theta) \), and therefore, conditional on exporting, firm \( \theta \) offers products of identical quality in the domestic and foreign markets. The latter is driven by the assumption of identical consumer preferences across countries.\(^{23}\)

In this case, the firm’s unit costs are also identical across markets and thus the optimal allocation of its total output must in turn equalize the marginal revenue of output in the domestic and foreign markets. From (4), this requires \( y_x(\theta)/y_d(\theta) \)\(^{1-\rho} = \tau^{-\rho}(A^*/A) \), which implies that the firm’s total revenue can be written as

\[
r(\theta) \equiv r_d(\theta) + I_x(\theta) r_x(\theta) = A q(\theta)^{\rho} y(\theta)^{\rho} \Upsilon(\theta)^{1-\rho},
\] (11)

where \( y(\theta) \) and \( q(\theta) \) are total output and product quality in firm \( \theta \), respectively. As in Helpman et al. (2010), the variable \( \Upsilon(\theta) \equiv 1 + I_x(\theta) [\tau^{-\rho}(A^*/A)]^{1/(1-\rho)} \) is a measure of market access for firm \( \theta \).

\(^{22}\)Intuitively, the firm can increase profits in a given market by either expanding output or quality. Optimality requires that choices of output and quality in each market satisfy the equality of relative marginal revenue and relative marginal cost. Note that variable trade costs increase (decrease) the marginal costs (revenues) of output and quality proportionally in the foreign market, thus they do not distort the relative marginal cost and the relative marginal revenue across markets. This property implies that the average cost of quality is independent of \( d \).

\(^{23}\)Quality upgrading induced by exporting can be easily introduced into the model by assuming that foreign consumers trade off quality and quantity differently than domestic consumers (see Verhoogen (2008)). For example, letting \( X^*_i = [ \int_{j \in J} (q^*(j) x^*_i(j))^{\chi+1} dj ]^{1/\chi} \), and \( \chi > 1 \). Alternatively, \( \chi < 1 \) induces domestic exporters to downgrade quality. This suggests that tastes in export destinations matter, as they may amplify or dampen the link between trade and inequality advanced in this paper. This extension is left for future versions of the paper.
Expression (8) implies that variable costs are equal to a fraction $\rho$ of revenue in each market. With identical quality across markets, the firm’s total profits can then be written as a fraction $1 - \rho$ of total revenue net of fixed costs,

$$\Pi(\theta) = (1 - \rho)r(\theta) - f_d - I_x(\theta) f_x.$$  \hfill (12)

As long as total revenue increases in firm productivity, the existence of a fixed production cost implies that there is a zero-profit cutoff $\theta_d$ such that firms drawing a productivity $\theta < \theta_d$ exit without producing. Similarly, the existence of a fixed exporting cost implies that there is a zero-profit cutoff $\theta_x$ such that $I_x(\theta) = 0$ if and only if $\theta < \theta_x$.24 This implies that the firm market access variable can be written as

$$\Upsilon(\theta) = \begin{cases} 
\Upsilon_x & \text{if } \theta \geq \theta_x, \\
1 & \text{if } \theta < \theta_x,
\end{cases}$$

where $\Upsilon_x \equiv 1 + e^{-\frac{1}{1-\rho} \left( A^*/A \right)^{\frac{1}{1-\rho}}} > 1$.

**Closed-form Solutions.** As shown in expression (9), product quality $q(\theta)$ is fully determined by the unit-cost function $c^\theta(\cdot)$. The following three functional form assumptions in turn determine $c^\theta(\cdot)$, guaranteeing existence, uniqueness and optimality of $q(\theta)$, and yielding closed-form solutions to the profit maximization problem:

$$k(\epsilon) = k \epsilon, \quad k > 0,$$

$$\mu(\epsilon) = -1/\epsilon,$$

$$\log q = (\gamma \log \theta)^z (1/n)^{(1-z)} , \quad \gamma > 0, \quad z \in (0, 1).$$

Note that (13) and (14) satisfy Assumption (1). By (14), (2) and Lemma 1, $n = -\mu(\epsilon) > 0$ almost surely.25 This guarantees that quality is properly defined in (15). Under specification (15), quality is log-submodular in $\theta$ and $n$ and thus the return to reducing the number of mistakes increases in firm productivity. This key property implies that high productivity firms have a comparative advantage in producing high quality output, which is consistent with the empirical evidence in Johnson (2012) and Kugler and Verhoogen (2012).26

Minimizing the average cost of quality under (13)-(15) yields a closed-form solution for a unique optimal quality $q(\theta)$ (see Appendix). With slight abuse of notation, optimal effort is in turn obtained from $q(\theta) = q(\theta, -\mu(\epsilon(\theta)))$. In the case of interior solutions,

$$q(\theta) = \theta^{\kappa_q}, \quad \kappa_q \equiv \gamma \left[ (1-z)/k \right]^{(1-z)/z},$$

$$\epsilon(\theta) = \kappa_\epsilon \log \theta, \quad \kappa_\epsilon \equiv \gamma \left[ (1-z)/k \right]^{1/z}.$$  \hfill (16) \hfill (17)

Note that $q(\theta)$ and $\epsilon(\theta)$ will in fact be interior for every firm $\theta \in [\theta_L, \theta_H]$ provided that individual effort is defined over a sufficiently large interval; that is, if $\kappa_\epsilon \left[ \log \theta_L, \log \theta_H \right] \subset \left[ \epsilon_L, \epsilon_H \right]$. For expositional ease, I will henceforth focus on this parameter configuration.27

---

24Note that $r(\theta)$ increases in $\theta$ if and only if $r_m(\theta)$ increases in $\theta$, for any $m = \{d, x\}$.

25Under (14), $\mu(\cdot) \in [\mu(\epsilon_L), \mu(\epsilon_H)]$, satisfying the requirements of Lemma 1.

26Formally, in this context, $q(\theta, N)$ is log-submodular if, for any $0 < \theta_0 < \theta_1$ and $0 < N_0 < N_1$, then

$$\log q(\theta_0, N_0) + \log q(\theta_1, N_1) > \log q(\theta_0, N_0) + \log q(\theta_1, N_1).$$

For an in-depth analysis of the relationship between supermodularity, submodularity and comparative advantage, see Costinot (2009).

27The model can deliver other, potentially interesting, types of equilibria. For example, if $\epsilon_L > \kappa_\epsilon \log \theta_d$ firms with sufficiently low productivity implement the minimum effort (and quality) by offering a flat wage that is independent of performance. In this equilibrium, only a fraction of the jobs in the economy are performance-pay jobs. Moreover, this fraction will typically decrease with trade liberalization, as long as a lower variable trade cost leads to a higher $\theta_d.$
From the first-order condition for output (8) and the expression for firm revenue (11), I solve for total output and revenue as functions of the demand shifters and the reservation utility. Total employment, denoted \( h(\theta) \), follows from the production function (6). Therefore:

\[
\begin{align*}
    r(\theta) &= \kappa_r \Upsilon(\theta) (A \pi^{1-\rho})^{1/(1-\rho)} \Gamma, \quad \kappa_r \equiv \rho^{\rho/(1-\rho)}, \\
    y(\theta) &= \kappa_y \Upsilon(\theta) (A \pi^{-1})^{1/(1-\rho)} \theta^{\Gamma + s - \kappa \varepsilon}, \quad \kappa_y \equiv \rho^{1/(1-\rho)}, \\
    h(\theta) &= \kappa_y \Upsilon(\theta) (A \pi^{-1})^{1/(1-\rho)} \theta^{\Gamma - \kappa \varepsilon},
\end{align*}
\]

where \( \Gamma \equiv \rho (\kappa_d - \kappa \varepsilon + s)/(1 - \rho) \). The condition \( \rho > 1 - z \) implies \( \Gamma > \kappa \varepsilon \), ensuring that revenue, output and employment increase in productivity for all \( s \geq 0 \). As usual in models with a fixed exporting cost and selection into export markets, firm revenue, output and employment increase discontinuously at the exporting cutoff as the marginal exporter incurs \( f_x \). This is not the case for quality and effort, since there is no motif for quality upgrading (or downgrading) associated to exporting in this model (see footnote in page 13).

### 4 Equilibrium

This section explains how to compute the remaining endogenous variables in the model. The focus is on analyzing symmetric equilibria, in which labor endowments, firm productivity distributions and trade costs are identical across countries.

The zero-profit cutoff \( \theta_d \) is the productivity level that makes firms indifferent between exiting and producing for the domestic market. In turn, the exporting cutoff \( \theta_x \) makes firms indifferent between exporting and producing exclusively for the domestic market. From the expressions for revenue (18) and profits (12), these two conditions require

\[
\kappa_r (1 - \rho) (\pi^{1-\rho})^{\rho/(1-\rho)} \theta^\Gamma_d E = f_d
\]

and

\[
\kappa_r (1 - \rho) (\pi^\rho)^{\rho/(1-\rho)} \theta^\Gamma_x E = f_x,
\]

respectively.\(^{28}\) Dividing (22) by (21) yields

\[
x^{-\rho}(\theta_x/\theta_d)^\Gamma = \frac{f_x}{f_d}.
\]

Free entry implies that the expected profit of successful entrants should equal the sunk entry cost; that is, \( \int_{\theta_d}^{\theta_x} \Pi(\theta) dG(\theta) = f_e \). Using the expressions for revenue (18) and productivity cutoffs (21) and (22), the free entry condition can be written as

\[
f_d J(\theta_d) + f_x J(\theta_x) = f_e,
\]

where \( J(\theta_m) \equiv \int_{\theta_m}^{\theta_x} \left[ (\theta/\theta_m)^\Gamma - 1 \right] dG(\theta), \ m = \{d, x\} \), is monotonically decreasing for any \( G(\theta) \).\(^{29}\)

\(^{28}\) In deriving these expressions, I use \( A = E(1-\rho) \), which follows by definition of the demand shifter \( A \) and the choice of numeraire \( (\pi = 1) \). Expression (22) also uses the fact that \( \Upsilon_x = 1 = e^{-\rho/(1-\rho)} \) in any symmetric equilibrium.

\(^{29}\) \( J \) is finite under the assumption that the distribution of firm productivity has a finite \( \Gamma \)-th uncentered moment.
Equations (23) and (24) fully determine the productivity cutoffs. For the remainder of the paper, I restrict the analysis to a class of equilibria satisfying \( \theta_L < \theta_d \leq \theta_x \). The condition \( \theta_L < \theta_d \) is necessary for the existence of equilibrium.\(^{30}\) As shown in the Appendix, given \( f_x, f_d, f_e, \Gamma \) and \( G(\theta) \), this condition holds provided that firm productivity is defined over a sufficiently large interval \([\theta_L, \theta_H]\). In turn, \( \theta_d \leq \theta_x \) is imposed for consistency with a large empirical literature documenting selection of the most productive firm into exporting. As in Melitz (2003), \( \theta_d \leq \theta_x \) if and only if \( (f_x/f_d)^{1-\rho} \geq 1 \). The analysis therefore includes closed economy equilibria (\( \theta_x \geq \theta_H \)) and equilibria in which all firms export (\( \theta_d = \theta_x \)).

Equilibrium in the differentiated goods market requires the equality of aggregate expenditure and aggregate revenue. The latter equals \( M \tau \), where \( M \) and \( \tau \) denote the mass and average revenue of active firms, respectively. Given a mass of entrants \( M_e \), the mass of active firms is \( [1 - G(\theta_d)] M_e \). In turn, the average revenue of producers can be written as a function of the productivity cutoffs by integrating firm profits (12) and using the free entry condition.\(^{31}\) The market clearing condition for the goods market can then be written as
\[
(1 - \rho) E = M_e [f_e + f_d [1 - G(\theta_d)] + f_x [1 - G(\theta_x)]]. \tag{25}
\]

Finally, labor market clearing requires equating labor supply, \( L \), and labor demand, \( M_e \int_{\theta_d}^{\infty} h(\theta) dG(\theta) \). Using expression (20) to substitute for firm employment yields
\[
L = \kappa_y (\pi)^{-1/(1-\rho)} E M_e \int_{\theta_d}^{\theta_H} \Upsilon(\theta) \theta^{\Gamma - k_k} dG(\theta). \tag{26}
\]

The general equilibrium with two symmetric countries is characterized by the productivity cutoffs \( \theta_d \) and \( \theta_x \), the aggregate expenditure \( E \), the mass of entrants \( M_e \) and the reservation utility \( \pi \). These equilibrium variables are obtained from equations (21), (22), (24), (25) and (26).

**Proposition 2** There exists a unique symmetric equilibrium.

**Proof.** Appendix. \( \blacksquare \)

## 5 Trade Liberalization, Selection and Inequality

This section studies the impact of trade liberalization on reallocations of workers and wage shares across firms, in symmetric equilibria. Throughout, I use subindex \( j \) to indicate equilibria before (\( j = 0 \)) and after (\( j = 1 \)) trade liberalization, respectively.

I analyze the effects of a decline in the (bilateral) variable trade cost, \( \iota_0 > \iota_1 \), for \( \iota_1 \in [\underline{\iota}, \bar{\iota}] \), holding the remaining parameters of the model constant. The lower bound for \( \iota_1 \) satisfies \( \underline{\iota} \equiv \max \left\{ 1, (f_d/f_x)^{(1-\rho)/\rho} \right\} \) and ensures that the equilibrium following trade liberalization features selection of the most productive firm into exporting. As a limit case, if \( f_d > f_x \) then \( \iota_1 = \underline{\iota} \) corresponds to an equilibrium in which every firm exports following trade liberalization. The supremum for \( \iota_1 \), denoted \( \bar{\iota} \), is the variable trade cost such that \( \theta_{x,1} = \theta_H \), implicitly defined by equations (23) and (24). Note that \( \iota_j \geq \bar{\iota} \) if and only if equilibrium \( j \) is autarky. The restriction \( \iota_1 < \bar{\iota} \) thus ensures that trade liberalization effectively opens the economy.

\(^{30}\)On the other hand, \( \theta_d < \theta_H \) is always satisfied. Otherwise, if \( \theta_H \leq \theta_d \leq \theta_x \), then the free entry condition would be violated.

\(^{31}\)In particular, integrating (12) and using the fact that, by free entry, average firm profits are equal to \( f_e/ [1 - G(\theta_d)] \), yields \( (1 - \rho) \pi = f_e/ [1 - G(\theta_d)] + f_d + f_x [1 - G(\theta_x)] / [1 - G(\theta_d)] \).
5.1 Reallocations of Labor and Wage Shares Across Firms

Trade liberalization leads to shifts in the distribution of firm productivity that trigger reallocations of labor towards high productivity firms. This section characterizes this effect. Since optimal performance-pay contracts differ across firms, labor reallocations have profound implications for the equilibrium distribution of wages in the economy, which are studied in the next section.

The mass of workers employed in firms with productivity lower than or equal to \( \theta \) is \( M \int_{\theta_d}^{\theta} h(\theta') dG_{\theta}(\theta' | \theta \geq \theta_d) \), for \( \theta \geq \theta_d \). In any equilibrium of the model, the distribution of employment across firms, denoted \( G_h(\theta) \), measures the fraction of workers employed in firms with productivity less than or equal to \( \theta \). Using the expression for firm employment (20),

\[
G_h(\theta) = \frac{\int_{\theta_d}^{\theta} \Upsilon(\theta') (\theta')^{\Gamma-k_{nc}} dG_{\theta}(\theta')}{\int_{\theta_d}^{\theta_H} \Upsilon(\theta') (\theta')^{\Gamma-k_{nc}} dG_{\theta}(\theta')}, \quad \text{for } \theta \in [\theta_d, \theta_H]. \tag{27}
\]

Importantly, \( G_h(\theta) \) depends on just two endogeneous variables; namely, the productivity cutoffs. This property enables an analytical characterization of changes in the distribution of employment in terms of changes in the productivity cutoffs across equilibria.\(^{32}\) For this purpose, let \( G_{h,j}, \theta_{d,j} \) and \( \theta_{x,j} \) denote the employment distribution and productivity cutoffs in an equilibrium with variable trade cost \( \iota_j \), respectively.

**Lemma 2** Let \( G_{h,0} \) and \( G_{h,1} \) denote employment distributions corresponding to equilibria before and after trade liberalization, respectively.

(a) If \( \iota_0 \geq \iota \), then \( G_{h,1} \) first-order stochastically dominates \( G_{h,0} \).

(b) If \( \iota_0 < \iota \), consider \( \tilde{\theta} \in [\theta_{x,0}, \theta_H] \):

- If \( G_{h,1}(\tilde{\theta}) \leq G_{h,0}(\tilde{\theta}) \), then \( G_{h,1} \) first-order stochastically dominates \( G_{h,0} \).
- If \( G_{h,1}(\tilde{\theta}) > G_{h,0}(\tilde{\theta}) \), then \( G_{h,1} \) intersects \( G_{h,0} \) once, from below, in \([\theta_{d,1}, \theta_H]\).

**Proof.** Appendix. □

Figure XX (coming soon) illustrates the two types of admissible changes in the employment distribution that result from a decline in variable trade costs. In either case, the result is reminiscent of the tendency towards higher concentration of workers in high productivity firms, following trade liberalization, inherent of models with firm heterogeneity and selection into exporting.\(^{33}\) Indeed, Lemma (2) implies that \( G_{h,1} \) second-order stochastically dominates \( G_{h,0} \) whenever trade liberalization reduces the mass of active firms.\(^{34}\) For a sharp characterization of the effect of trade liberalization on within-firm inequality, however, I will rely on a stronger form of labor reallocation, obtained by imposing additional structure on the distribution of firm productivity.

\(^{32}\)In symmetric equilibria, productivity cutoffs respond to trade liberalization as in Melitz (2003). From (23), a decline in the variable trade cost \( \iota \) leads to a lower \( \theta_x/\theta_d \). Moreover, by the free entry condition (24) and the monotonicity of \( J(\cdot) \), the productivity cutoffs are inversely related. It follows immediately that trade liberalization leads to the exit of the least productive firms (higher \( \theta_d \)) and to the entry of new exporters to the foreign market (lower \( \theta_x \)).

\(^{33}\)In Melitz (2003), for example, trade liberalization leads to higher employment in exporting firms.

\(^{34}\)To see this, note that the single-crossing property stated in part (ii) of Lemma (2) is equivalent to second-order stochastic dominance if mean firm employment increases following trade liberalization (see, for example, Proposition 4.6 in Wolfstetter (1999)). Because the labor market clears, the latter condition holds if and only if trade liberalization reduces the mass of active firms.
Assumption 2 For $\theta \in [\theta_L, \theta_H]$, 

$$\frac{J'(\theta)}{g_0(\theta)} \text{ is non-decreasing in } \theta.$$ 

Assumption (2) is satisfied by a class of productivity distributions that includes Pareto, distributions with non-decreasing densities and, more generally, densities with elasticity greater than or equal to $-(\Gamma + 1)$ (see Section A.8 in the Appendix). Assumption (2) is sufficient to establish that $G_{h,1}(\theta) \leq G_{h,0}(\theta)$, for any $\theta \in [\theta_{x,0}, \theta_H]$. In light of Lemma (2), I obtain the following result.

**Proposition 3** Let $G_{h,0}$ and $G_{h,1}$ denote employment distributions corresponding to equilibria before and after trade liberalization, respectively. If $\iota_0 < \tau$, impose Assumption (2). Then $G_{h,1}$ first-order stochastically dominates $G_{h,0}$. That is, for all $\theta \in [\theta_L, \theta_H]$,

$$G_{h,1}(\theta) \leq G_{h,0}(\theta), \text{ with strict inequality for some } \theta.$$ 

**Proof.** Appendix.

Next, I analyze the impact of trade liberalization on the distribution of wages across firms. The wage bill paid by firms with productivity lower than or equal to $\theta$ is $L \int_{\theta_d}^{\theta} \omega(\theta')dG_h(\theta')$, for $\theta \geq \theta_d$. The distribution of wages across firms, denoted $G_w(\theta)$, measures the fraction of wages paid by firms with productivity less than or equal to $\theta$. Therefore,

$$G_w(\theta) = \frac{\int_{\theta_d}^{\theta} \omega(\theta')dG_h(\theta')}{\int_{\theta_d}^{\theta_H} \omega(\theta')dG_h(\theta')}, \text{ for } \theta \in [\theta_d, \theta_H]. \quad (28)$$

**Proposition 4** Let $G_{w,0}$ and $G_{w,1}$ denote wage distributions across firms corresponding to equilibria before and after trade liberalization, respectively. If $\iota_0 < \tau$, impose Assumption (2). Then $G_{w,1}$ first-order stochastically dominates $G_{w,0}$. That is, for all $\theta \in [\theta_L, \theta_H]$,

$$G_{w,1}(\theta) \leq G_{w,0}(\theta), \text{ with strict inequality for some } \theta.$$ 

**Proof.** Appendix.

### 5.2 Wage Inequality

There are two sources of heterogeneity in individual wages, a firm-specific component $\theta$ and a worker-specific component $B_i = B_i(1)$. The distribution of wages in the economy (and thus measures of wage inequality) will therefore depend on the underlying distributions of firm productivity $\theta$ and idiosyncratic performance $B$.

More specifically, combining the firm’s optimal choice of effort (17) with parts (a) and (b) of Proposition (1) and functional forms (13) and (14), yields the wage of worker $i$ employed in firm $\theta$,

$$w_i = w(\theta, B_i) = \overline{\pi} \exp \left[ k(\theta) + \beta(\theta) \left( B_i - \beta(\theta) / 2 \right) \right]. \quad (29)$$

For any $w_0$, let $\Phi(B(\theta, w_0))$ denote the fraction of employees in firm $\theta$ with wages lower than or equal to $w_0$, where $\Phi$ is the standard normal c.d.f. and $B(\theta, w_0)$ satisfies $w_0 = w(\theta, B(\theta, w_0))$. Then the wage distribution, denoted $F_w(w)$, is given by

$$F_w(w) = \int_{\theta_d}^{\theta_H} \Phi(B(\theta, w))dG_h(\theta). \quad (30)$$
The distribution of wages is therefore a mixture of the distributions of \( \theta \) and \( B \).

For concreteness, I start by analyzing a specific inequality measure constructed from (30), the variance of log wages.\(^{35}\) The latter has been frequently applied in recent empirical studies of wage inequality.\(^{36}\) Unlike other popular measures of inequality such as the Gini coefficient and the 90-10 wage gap, the variance is decomposable into between- and within-firm components. This property is analytically convenient to highlight different channels through which international trade impacts wage inequality. At the end of the section, however, I verify the robustness of the results by analyzing two additional, decomposable measures of inequality, the Theil index and the mean log deviation.

The following Proposition anticipates the results derived in this section and constitutes the main result of the paper. The theory developed in the previous sections delivers a sharp link between international trade liberalization and within-firm inequality. As in Helpman et al. (2010) and Coçar et al. (forthcoming), however, the effects on between-firm inequality are non-monotonic and difficult to characterize analytically without further assumptions.

**Proposition 5** Consider any symmetric equilibrium with variable trade cost \( \tau_0 \). If \( \tau_0 < \tau \), impose Assumption (2). Then a trade liberalization \( \tau_1 < \tau_0 \), for \( \tau_1 \in [\underline{\tau}, \overline{\tau}) \), leads to an increase in within-firm wage inequality, according to the following inequality measures:

- Variance of log wages
- Theil Index
- Mean Log Deviation

### 5.2.1 The Variance of Log Wages

In the model, different firms select different performance-pay contracts to reward their employees. This implies that within-firm wage distributions differ across firms, and thus inequality measures will crucially depend on the equilibrium allocation of workers across firms. In particular, the variance of log wages depends on the employment distribution and on the mean and variance of the within-firm log wage distributions, denoted \( E(\tilde{w}_i|\theta) \) and \( Var(\tilde{w}_i|\theta) \), respectively, where \( \tilde{w}_i \equiv \log w_i \). Given \( G_h(\theta) \), these two moments can be integrated across firms to obtain the standard decomposition of the total variance of log wages into between- and within-firm components. This yields,

\[
Var_j(\tilde{w}_i) = Var_j^{\text{between}}(\tilde{w}_i) + Var_j^{\text{within}}(\tilde{w}_i),
\]

where

\[
Var_j^{\text{between}}(\tilde{w}_i) = \int_{\theta_L}^{\theta_H} \left[ E(\tilde{w}_i|\theta) - \tilde{w}_j^* \right]^2 dG_{h,j}(\theta),
\]

\[
Var_j^{\text{within}}(\tilde{w}_i) = \int_{\theta_L}^{\theta_H} Var(\tilde{w}_i|\theta) dG_{h,j}(\theta),
\]

and \( \tilde{w}_j^* \equiv \int_{\theta_L}^{\theta_H} E(\tilde{w}_i|\theta) dG_{h}(\theta) \) is the mean log wage in equilibrium \( j \).

\(^{35}\)The logarithmic transformation ensures that this measure of inequality is invariant to proportional shifts in the wage distribution, e.g. changes in the reservation utility \( \bar{u} \) in equation (29).

\(^{36}\)For example, among recent empirical studies, Lemieux (2006), Helpman et al. (2012) and Card et al. (2013) use variance decompositions of log wages to analyze changes in inequality in the US, Brazil and Germany, respectively.
The between-firm variance is the variance of average log wages across firms, while the within-firm variance is the weighted average of firm-level variances. I will refer to the within-firm variance interchangeably as the residual variance of log wages. This label prevents confusion with the firm-level variances $\text{Var}(w_i|\theta)$ and also highlights a link between the analysis in this section and empirical studies of trade and inequality. For example, in Helpman et al. (2012), the between-firm component is the estimated variance of the firm-fixed effects in a regression of individual wages that also controls for observable worker characteristics. The within-firm component is the variance of the regression residuals.

As in previous related literature, wage inequality across ex-ante identical workers in the model is partly driven by cross-firm variation in average wages; i.e., between-firm inequality. Earlier models have shown that this variation can be generated by search frictions, efficiency wages or fair wage considerations, while in this model firms compensate their workers for exerting costly effort.

Unlike other models in the literature, however, part of the wage variation arises from differences in firm-level wage inequality across firms. As long as worker performance is only a noisy signal of effort, firms deal with the moral hazard problem by paying for performance. This implies $\text{Var}(\tilde{w}_i|\theta) > 0$ for all active firms; i.e., within-firm wage dispersion. Moreover, firm-level wage variances vary across firms. High productivity firms offer higher-powered incentives that magnify idiosyncratic differences in performance, translating it into higher wage inequality among co-workers. In particular, part (b)(i) of Corollary (1) and the expression for optimal effort (17) imply that $\text{Var}(\tilde{w}_i|\theta)$ increases in firm productivity even when the variance of idiosyncratic performance is identical in every firm. In the absence of quality upgrading associated to exporting, however, firm-level wage distributions are independent of the variable trade cost $\tau$. In this case, cross-firm variation in inequality is a necessary ingredient for trade liberalization to have an impact on within-firm inequality.

Next, I show that, in combination with the stronger form of labor reallocations implied by Assumption (2), this mechanism generates increasing within-firm wage inequality. The change in the between-firm variance, however, cannot be signed without imposing more structure on the distribution of firm productivity.

Formally, let subscripts 0 and 1 denote outcomes corresponding to equilibria before and after trade liberalization, respectively. Consider first the change in the residual variance, which can be written as

$$
\Delta \text{Var}_{\text{within}}(\tilde{w}_i) = \int_{\theta_L}^{\theta_H} \text{Var}(\tilde{w}_i|\theta) \left[ dG_{h,1}(\theta) - dG_{h,0}(\theta) \right],
$$

$$
= \int_{\theta_L}^{\theta_H} \frac{d\text{Var}(\tilde{w}_i|\theta)}{d\theta} \left[ G_{h,0}(\theta) - G_{h,1}(\theta) \right] d\theta,
$$

$$
> 0.
$$

The first line uses the fact that, in any equilibrium of the model, the firm-level wage distributions are independent of the variable trade cost. The second line follows after integrating by parts. Part (b)(i) of Corollary (1) and the expression for optimal effort (17) imply that the firm-level variance increases in $\theta$, thus $d\text{Var}(\tilde{w}_i|\theta)/d\theta > 0$. Moreover, Proposition (3) implies $G_{h,0}(\theta) \geq G_{h,1}(\theta)$ for all $\theta$, with strict inequality for some $\theta$. Intuitively, under Assumption (2), trade liberalization generates strong labor reallocations towards high inequality firms, resulting in an unambiguous increase in the residual variance of log wages.\(^{37}\)

\(^{37}\)Recall that, by Lemma (2), Assumption (2) is not needed when the initial equilibrium is autarky.
In turn, the change in the between-firm variance is given by

$$
\Delta \text{Var}_{j}^{\text{between}}(\tilde{w}_i) = \int_{\theta_L}^{\theta_H} [E(\tilde{w}_i|\theta)]^2 [dG_{h,1}(\theta) - dG_{h,0}(\theta)] - [(\tilde{w}_1^*)^2 - (\tilde{w}_0^*)^2],
$$

$$
= 2 \int_{\theta_L}^{\theta_H} \frac{dE(\tilde{w}_i|\theta)}{d\theta} E(\tilde{w}_i|\theta) [G_{h,0}(\theta) - G_{h,1}(\theta)] d\theta - [(\tilde{w}_1^*)^2 - (\tilde{w}_0^*)^2].
$$

As in the analysis of the residual variance, the second line is obtained after integrating by parts. However, the change in $\text{Var}_{j}^{\text{between}}(\tilde{w}_i)$ cannot, in general, be signed. Note that the mean log wage is not necessarily increasing in firm productivity.\(^{38}\) But even if it were, labor reallocations towards high productivity firms would then imply a rise in the mean log wage, $\tilde{w}_1^* > \tilde{w}_0^*$, that tends to reduce the between-firm variance in the aftermath of trade liberalization.

### 5.2.2 Lorenz-consistent Inequality Measures

Similar results follow from the analysis of alternative inequality measures. Although the variance of log wages is a popular measure for inequality comparisons in applied work, it may conflict with the Lorenz criterion (Foster and Ok (1999)).\(^{39}\) The latter, however, incorporates some principles that are generally regarded as fundamental to the theory of inequality measurement.\(^{40}\) For this reason, I close this section by analyzing the impact of trade liberalization on two Lorenz-consistent measures, the mean log deviation ($\text{MLD}$) and the Theil index ($T$). These inequality measures, introduced by Theil (1967), belong to the generalized entropy class and, as such, they can be decomposed into between and within components.\(^{41}\)

The definition and decomposition of the MLD and Theil measures in equilibrium $j \in \{0, 1\}$ are given by

$$
\text{MLD}_j \equiv E_j \left[ \log \left( \frac{w_i^*}{\tilde{w}_i} \right) \right],
$$

$$
= \int_{\theta_L}^{\theta_H} \log \left( \frac{w_i^*}{\omega(\theta)} \right) dG_{h,j}(\theta) + \int_{\theta_L}^{\theta_H} \text{MLD} \left( w_i|\theta \right) dG_{h,j}(\theta), \quad (31)
$$

\(^{38}\)Intuitively, expected firm-level wages increase in productivity but so does wage dispersion. These two forces operate in opposite directions on average log wages, since the log transformation is both increasing and concave. When productivity is high enough, the mean log wage decreases in $\theta$.

\(^{39}\)The Lorenz criterion states that a distribution $F$ is more unequal that distribution $F'$ if and only if the Lorenz curve of $F$ lies below the Lorenz curve of $F'$ everywhere in the domain.

\(^{40}\)Atkinson (1970) showed that this criterion is equivalent to second-order stochastic dominance when the two distributions have equal mean.

\(^{41}\)Generalized entropy measures have several desirable properties. Theorem 5 in Shorrocks (1980) shows that an inequality measure simultaneously satisfies the weak principle of transfers, decomposability, scale independence and the population principle only if it belongs to the class of generalized entropy measures. Moreover, as pointed out in Shorrocks (1980), $\text{MLD}$ and $T$ enjoy two analytical advantages relative to any other generalized entropy measure. First, the total within-firm contribution to inequality is a weighted average of inequality across firms only for $\text{MLD}$ and $T$. Second, the decomposition coefficients are independent of the between-group contribution only for $\text{MLD}$ and $T$. 

21
and

\[
T_j = E_j \left[ \log \left( \frac{w_i}{w_j^*} \right) \frac{w_i}{w_j^*} \right],
\]

\[
= \int_{\theta_L}^{\theta_H} \log \left( \frac{\omega(\theta)}{w_j^*} \frac{\omega(\theta)}{w_j^*} \right) dG_{h,j}(\theta) + \int_{\theta_L}^{\theta_H} T(w_i|\theta) dG_{w,j}(\theta),
\]

(32)

respectively, where \( w_j^* = \int_{\theta_L}^{\theta_H} E(w_i|\theta)dG_{h,j}(\theta) \) is the mean wage in equilibrium \( j \).

Expressions (31) and (32) show that the MLD and Theil indices can be decomposed into a component measuring inequality of mean wages across firms (between-firm inequality) and a component measuring average firm-level inequality (within-firm inequality). Importantly, within-firm inequality has a similar structure in both measures, which also resembles the structure of the residual variance of log wages. In particular, for measure \( I = \{MLD, T\} \),

\[
I_{within}^j = J_{\theta(j)} \int_{\theta_L}^{\theta_H} I(w_i|\theta)dG_{\ell(I),j}(\theta),
\]

where \( \ell(I) = h \) if \( I = MLD \) and \( \ell(I) = w \) if \( I = T \).

The impact of trade liberalization on \( I_{within} \) can then be evaluated as in the case of \( Var_{within} \). In particular,

\[
\Delta I_{within} = \int_{\theta_L}^{\theta_H} I(w_i|\theta) \left[ dG_{\ell(I),1}(\theta) - dG_{\ell(I),0}(\theta) \right],
\]

\[
= \int_{\theta_L}^{\theta_H} \frac{dI(w_i|\theta)}{d\theta} \left[ G_{\ell(I),0}(\theta) - G_{\ell(I),1}(\theta) \right] d\theta,
\]

\[
> 0.
\]

This result relies on part (b)(ii) of Corollary (1), which ensures \( dI(w_i|\theta)/d\theta > 0 \). Moreover, Propositions (3) and (4) imply \( G_{\ell(I),0}(\theta) \geq G_{\ell(I),1}(\theta) \) for \( I = \{MLD, T\} \) and all \( \theta \), with strict inequality for some \( \theta \).

As anticipated, the effect of trade liberalization on between-firm inequality cannot be signed without further assumptions. Still, some progress can be made under the assumption that productivity follows an unbounded Pareto distribution. In this case, it is straightforward to verify that between-firm inequality in the open economy when all firms export is the same as in autarky, according to both the MLD and T measures. This result is reminiscent of Proposition (3)(ii) in Helpman et al. (2010).

6 Concluding Remarks

Evidence collected from matched employer-employee data consistently shows that wage dispersion within firms is a major component of wage inequality in many countries. This paper is, to the best of my knowledge, the first in the literature to develop a general equilibrium framework to study the determinants of within-firm wage dispersion and its links to international trade liberalization. Moreover, in light of the magnitude and growth of residual wage dispersion, the focus is on modeling within-firm wage inequality between identical workers. Trade liberalization triggers reallocations of workers towards firms that intensively rely on
contracting strategies that generate higher wage dispersion among co-workers. The paper identifies conditions under which this mechanism delivers a sharp analytical characterization of the effect of trade liberalization on within-firm inequality.

Motivated by the empirical evidence documenting the prevalence and growth of performance pay in the U.S., I have emphasized heterogeneity in optimal performance-pay contracts as the key source of within-firm inequality in the model. In doing so, I have abstracted from cross-firm differences in the composition of worker skills that, if unobservable to the econometrician, would constitute an additional source of residual within-firm inequality in the data. Needless to say, I do so for analytical convenience and because (I conjecture that) introducing differences in the composition of workers in the analysis is most likely to operate as a complementary source of within-firm inequality. In such an extension of the model, a firm would design a contract for each type of worker. Wage dispersion within a firm would then be composed of wage variation between and within worker types. In this context, the theory developed in this paper can be interpreted as a mechanism generating within-firm wage dispersion among workers of a given type.

A common feature in several studies in the literature, yet absent in this framework, are exporter wage premia. In the model, conditional on productivity, exporting does not induce firms to pay higher wages. As mentioned, however, one way in which this feature can be incorporated into the model is by assuming that foreign buyers have a relatively higher preference for quality than domestic consumers, as in Verhoogen (2008). Alternatively, introducing increasing marginal costs of output may lead exporters and non-exporters to upgrade and downgrade quality, respectively, as a result of trade liberalization. These extensions would also generate higher within-firm inequality in exporting firms, conditional on productivity, which is consistent with the empirical evidence reported in Frías et al. (2012). Importantly, my analysis shows that exporter wage premia are not necessary for international trade to have an impact on within-firm wage inequality. Introducing exporter wage premia is likely reinforce the main results of the paper.

There are a number of additional topics worth exploring in future versions of this draft. First, the impact of trade liberalization on ex-post welfare. On one hand, lower trade costs lead to lower consumption prices and higher expected wages. However, labor reallocations towards high productivity firms can potentially hurt unlucky workers who, despite high effort levels, end up receiving low wages due to poor ex-post performance. Second, although the hypothesis that quality depends on employee performance appears to be a natural assumption, vertical differentiation may not be an indispensable part of the mechanism linking trade liberalization to wage inequality advanced in this paper. Developing alternative settings that lead to cross-firm heterogeneity in performance pay would constitute an important extension of the present framework.

References


A Appendix

A.1 Proof of Lemma 1

For any worker $i$ and task duration $\Delta \equiv 1/T$, let $\{\mathcal{F}^\Delta_{i\tau}\}_{\tau=0}^T$ be a sequence of $\sigma$-fields on the underlying probability space, where $\mathcal{F}^\Delta_{i0}$ is the trivial $\sigma$-field and $\mathcal{F}^\Delta_{i1},...,\mathcal{F}^\Delta_{iT}$ is the filtration generated by the random variables $z_{i1},...,z_{iT}$. Let $Z^\Delta_{i\tau}$ denote the (normalized) cumulative performance of worker $i$ up to task $\tau \in \{1,...,T\}$ with initial condition $z_{i0} = 0$; i.e., $Z^\Delta_{i\tau} \equiv \Delta^{1/2} \sum_{\tau'=1}^\tau z_{i\tau'}$.\footnote{Throughout, I adhere to the convention that the value of an empty sum of numbers is zero. Thus, for example, $Z^\Delta_{i0} = 0$.} Let $\overline{z_{i\tau}} \equiv E \left( z_{i\tau} | \mathcal{F}^\Delta_{i\tau-1} \right) = \mu(\epsilon^\Delta_{i\tau}) \Delta^{1/2}$ for $\tau \in \{1,...,T\}$, where effort is allowed to be history-dependent; i.e., $\epsilon^\Delta_{i\tau} = \epsilon^\Delta_{i\tau}(z_{i1},...,z_{i\tau-1})$. Then, for $\tau \in \{0,...,T\}$,

$$Z^\Delta_{i\tau} \equiv \Delta \sum_{\tau'=1}^\tau \mu(\epsilon^\Delta_{i\tau'}) + B^\Delta_{i\tau}, \quad (A-1)$$

where

$$B^\Delta_{i\tau} \equiv \Delta^{1/2} \sum_{\tau'=1}^\tau (z_{i\tau'} - \overline{z_{i\tau'}})$$

denotes the cumulative deviation of worker $i$’s performance from its expected value up to task $\tau$. Equation (A-1) is an identity, by definitions of $B^\Delta_{i\tau}$ and $Z^\Delta_{i\tau}$. For the remainder of the proof, I suppress the subscript $i$ to simplify notation.

Let $Z^\Delta(t), t \in [0,1]$, be the piecewise linear interpolation defined in equation (1). Then, using equation (A-1),

$$Z^\Delta(t) = \left(1 - \frac{t}{\Delta} + \left\lfloor \frac{t}{\Delta} \right\rfloor \right) \left[ \Delta \sum_{\tau'=1}^{\lfloor t/\Delta \rfloor} \mu(\epsilon^\Delta_{i\tau'}) + B^\Delta_{[t/\Delta]} \right] + \left(\frac{t}{\Delta} - \left\lfloor \frac{t}{\Delta} \right\rfloor \right) \left[ \Delta \sum_{\tau'=1}^{\lfloor t/\Delta \rfloor + 1} \mu(\epsilon^\Delta_{i\tau'}) + B^\Delta_{[t/\Delta]+1} \right],$$

$$= \Delta \sum_{\tau'=1}^{\lfloor t/\Delta \rfloor} \mu(\epsilon^\Delta_{i\tau'}) + B^\Delta(t) + \left(\frac{t}{\Delta} - \left\lfloor \frac{t}{\Delta} \right\rfloor \right) \Delta b(\epsilon^\Delta_{[t/\Delta]+1}),$$

$$= \Delta \sum_{\tau'=1}^{\lfloor t/\Delta \rfloor} \mu(\epsilon^\Delta_{i\tau'}) + B^\Delta(t) + o(\Delta),$$

$$= \int_{\lfloor t/\Delta \rfloor \Delta}^{\lceil t/\Delta \rceil \Delta} \mu(\epsilon^\Delta(t')) dt' + B^\Delta(t) + o(\Delta). \quad (A-2)$$

In the third line, $B^\Delta(t)$ is the piecewise linear interpolation between the points $(0,0), (\Delta, B^\Delta_1), (2\Delta, B^\Delta_2),..., (1, B^\Delta_T)$. More specifically,\footnote{This interpolation is equivalent to that employed in Brown (1971). Unlike the definition in equation (A-3), the procedure in Brown (1971) includes adjustments by the martingale’s variance at different points of the process. For the case considered in this paper, however, $E \left( \epsilon^\Delta_{\tau} \right)^2 = \Delta \tau$ for all $\tau$, by equation (A-8). It is then straightforward to show that the interpolation of $\left\{ \epsilon^\Delta_{\tau} \right\}_{\tau=0}^T$ following Brown’s procedure is equivalent to equation (A-3).}

$$B^\Delta(t) = \left(1 - \frac{t}{\Delta} + \left\lfloor \frac{t}{\Delta} \right\rfloor \right) B^\Delta_{[t/\Delta]} + \left(\frac{t}{\Delta} - \left\lfloor \frac{t}{\Delta} \right\rfloor \right) B^\Delta_{[t/\Delta]+1}, \quad (A-3)$$
In the fourth line above, \( o(\Delta) \to 0 \) a.s. as \( \Delta \to 0 \), which follows from the boundedness of \( \mu(\cdot) \) and the fact that \( (t/\Delta - \lfloor t/\Delta \rfloor) \in [0,1) \). Finally, the fifth line introduces a change of integration variables, \( t' = \Delta \tau' \), and a continuous-time representation \( \epsilon^\Delta(t) \), where \( \epsilon^\Delta(t) \equiv \epsilon^\Delta_{\lfloor t/\Delta \rfloor} \).

Next, I characterize the convergence of each term on the right-hand side of equation (A-2), as \( \Delta \to 0 \). Regarding the first term, \( \epsilon^\Delta(t) \to \epsilon(t) \) a.s. as \( \Delta \to 0 \) and the continuity of \( \mu(\cdot) \) imply that \( \mu(\epsilon^\Delta(t)) \to \mu(\epsilon(t)) \) a.s. as \( \Delta \to 0 \), by the continuous mapping theorem. Moreover, since \( \mu(\cdot) \) is bounded, the bounded convergence theorem implies

\[
\int_\Delta \Delta \mu(\epsilon^\Delta(t'))dt' \to \int_0^t \mu(\epsilon(t'))dt' \text{ a.s. as } \Delta \to 0,
\]

using the fact that \( |t/\Delta| \Delta \to t \) as \( \Delta \to 0 \).

Regarding the limit of the second term on the right-hand side of (A-2), I start by noting two properties of \( B^\Delta_t \). First, for all \( s, \tau \in \{0, ..., T\} \) such that \( s < \tau \), applying the law of iterated expectations yields

\[
E \left( B^\Delta_t | F^\Delta_s \right) = \Delta^{1/2} \sum_{\tau'=1}^\tau E \left( z_{\tau'} - \tau_{\tau'} | F^\Delta_s \right)
\]

\[
= \Delta^{1/2} \sum_{\tau'=1}^\tau (z_{\tau'} - \tau_{\tau'}) + \Delta^{1/2} \sum_{\tau'=s+1}^\tau \left[ E \left( z_{\tau'} | F^\Delta_s \right) - E \left( z_{\tau'} | F^\Delta_{\tau-1} \right) \right]
\]

\[
= B^\Delta_s. \tag{A-5}
\]

Second, for any \( \tau \), \( E \left[ (z_{\tau} - \tau_{\tau})^2 | F^\Delta_{\tau-1} \right] = 1 + (z_{\tau})^2 - \tau E \left[ z_{\tau} | F^\Delta_{\tau-1} \right] = 1. \) This implies

\[
\sum_{\tau=1}^T E \left[ (B^\Delta_t - B^\Delta_{\tau-1})^2 | F^\Delta_{\tau-1} \right] = E \left[ \sum_{\tau=1}^T E \left[ (B^\Delta_t - B^\Delta_{\tau-1})^2 | F^\Delta_{\tau-1} \right] \right] \text{ a.s., for all } \Delta. \tag{A-6}
\]

Conditions (A-5) and (A-6) imply that, for given \( \Delta \), the process \( \{B^\Delta_t\}_{\tau=0}^T \) belongs to the class of zero-mean, square-integrable martingales relative to \( \{F^\Delta_{\tau}\}_{\tau=0}^T \) studied in Brown (1971). In particular, Theorem 3 in Brown (1971) implies that, as \( \Delta \to 0 \), the sequence of probability measures determined by the distribution of \( \{B^\Delta(t); 0 \leq t \leq 1\} \) converges weakly to the Wiener measure in the space \( C[0,1] \) with the uniform norm, provided that the Lindeberg condition holds, namely

\[
E \left[ (B^\Delta_t)^2 \right]^{-2} \sum_{\tau=1}^T E \left[ \Delta (z_{\tau} - \tau_{\tau})^2 I \left( \Delta^{1/2} | z_{\tau} - \tau_{\tau} | \geq \delta E \left[ (B^\Delta_t)^2 \right] \right) \right] \to_p 0, \tag{A-7}
\]

44Note that \( b(\cdot) \) is a continuous function defined on the closed interval \( [\epsilon_L, \epsilon_H] \), which ensures that \( b(\cdot) \) is also bounded.

45Brown (1971) considers zero-mean, square-integrable martingales \( \{\epsilon^\Delta_{\tau}\}_{\tau=0}^T \) that satisfy

\[
\frac{\sum_{\tau=1}^T E \left[ (\epsilon^\Delta_{\tau} - \epsilon^\Delta_{\tau-1})^2 | F^\Delta_{\tau-1} \right]}{E \left[ \sum_{\tau=1}^T E \left[ (\epsilon^\Delta_{\tau} - \epsilon^\Delta_{\tau-1})^2 | F^\Delta_{\tau-1} \right] \right]} \to_p 1,
\]

as \( \Delta \to 0 \), which is implied by (A-6).
as $\Delta \to 0$, for all $\delta > 0$, where $I(\cdot)$ is the indicator function.

To check (A-7), first note that $E\left[\left(B^2_\tau\right)^2\right] = 1$ for all $\Delta$, since martingale increments are uncorrelated\footnote{That is, for $\tau_1 > \tau_0 \geq 0$, let $s_{10} = E\left[\left(z_{\tau_0} - \overline{z}_{\tau_0}\right)\left(z_{\tau_1} - \overline{z}_{\tau_1}\right)\right]$. Then, applying the law of iterated expectations twice yields $s_{10} = E\left[\left(z_{\tau_0} - \overline{z}_{\tau_0}\right) E\left[\left(z_{\tau_1} - \overline{z}_{\tau_1}\right) | \mathcal{F}_{\tau_0}\right]\right]$, $= E\left[\left(z_{\tau_0} - \overline{z}_{\tau_0}\right) E\left[\left(z_{\tau_1} - \overline{z}_{\tau_1}\right) | \mathcal{F}_{\tau_0}\right]\right]$, $= 0$.} and thus

$$
E\left[\left(B^2_\tau\right)^2\right] = \Delta E\left[\sum_{\tau'=1}^{\tau} \left(z_{\tau'} - \overline{z}_{\tau'}\right)^2\right],
$$

$\quad = \Delta E\left[\sum_{\tau'=1}^{\tau} \left(z_{\tau'} - \overline{z}_{\tau'}\right)^2\right],
$$

$\quad = \Delta E\left[\sum_{\tau'=1}^{\tau} E\left[\left(z_{\tau'} - \overline{z}_{\tau'}\right)^2 | \mathcal{F}_{\tau'-1}\right]\right],
$$

$\quad = \Delta E\left[\sum_{\tau'=1}^{\tau} \left[1 + \left(z_{\tau'} - \overline{z}_{\tau'}\right)^2 - \overline{z}_{\tau'} E\left[\left(z_{\tau'} | \mathcal{F}_{\tau'-1}\right)\right]\right],
$$

$\quad = \Delta \tau. \quad (A-8)$

Second, let $\mu_H = \sup_x |\mu(x)| < \infty$, since $\mu(\cdot)$ is bounded. Therefore, for any $\tau$,

$$
\Delta^{1/2} |z_\tau - \overline{z}_\tau| < \Delta^{1/2} \left(1 + \Delta^{1/2} \mu_H\right) \to 0, \quad \text{as} \; \Delta \to 0.
$$

Letting $K^\Delta$ denote the left-hand side of (A-7) yields

$$
K^\Delta = \sum_{\tau=1}^{T} E\left[\Delta \left(z_\tau - \overline{z}_\tau\right)^2 I\left(\Delta^{1/2} |z_\tau - \overline{z}_\tau| \geq \delta\right)\right],
$$

$\quad < I\left(\Delta^{1/2} \left(1 + \Delta^{1/2} \mu_H\right) \geq \delta\right) \sum_{\tau=1}^{T} E\left[\Delta \left(z_\tau - \overline{z}_\tau\right)^2\right],
$$

$\quad = I\left(\Delta^{1/2} \left(1 + \Delta^{1/2} \mu_H\right) \geq \delta\right).

Therefore, $K^\Delta \to 0$ a.s. as $\Delta \to 0$, which is sufficient to verify the Lindeberg condition (A-7). Therefore, by Theorem 3 in Brown (1971),

$$
B^\Delta(t) \to^d B(t), \quad (A-9)
$$

in the space $C[0, 1]$ with the uniform norm, where $B(t)$ is a Wiener process on $0 \leq t \leq 1$, such that $B(0) = 0 \text{ a.s.}$ and $E\left[B(1)^2\right] = 1$.

Applying the results (A-4) and (A-9) to equation (A-2), I conclude that, as $\Delta \to 0$, $Z^\Delta(t)$ converges in distribution to a stochastic process $Z(t)$ in $C[0, 1]$, such that

$$
Z(t) = \int_{0}^{t} \mu(\epsilon(t')) dt' + B(t),
$$

which completes the proof.
A.2 Proof of Proposition 1

The proof proceeds in three steps:

**Step 1.** From the first-order conditions of worker $i$’s problem, I show that if a contract $w_i(Z_i^1)$ implements the stochastic process $\epsilon_i \equiv \{\epsilon_i(t); t \in [0, 1]\}$ with certain equivalent $\chi_i$, then

$$
\ln w_i(Z_i^1) = \ln \chi_i + \int_0^1 \frac{k'(\epsilon_i(t))}{\mu'(\epsilon_i(t))} [dZ_i(t) - \mu(\epsilon_i(t))dt] - \frac{1}{2} \int_0^1 \left[ \frac{k'(\epsilon_i(t))}{\mu'(\epsilon_i(t))} \right]^2 dt. 
$$

(A-10)

I start by introducing a change of variables, letting $s_i(Z_i^1) = \ln w_i(Z_i^1)$, to re-write the problem of worker $i$ -constraint (iii) in Problem (5)- as

$$
\max_{\epsilon_i} E \left[ \exp \left( s_i(Z_i^1) - \int_0^1 c(\epsilon_i(t'))dt' \right) \right] 
$$

(A-11)

$$
s.t \quad Z_i(t) = \int_0^t \mu (\epsilon_i (t')) dt' + B_i(t). 
$$

Formulated in this way, the worker’s problem is similar to that in Holmström and Milgrom (1987), although they work with negative exponential utility -CARA- and set $\mu(x) = x$. I will therefore modify the proof of Theorem 6 in Holmström and Milgrom (1987) allowing for positive exponential utility and a general (differentiable) function $\mu(\cdot)$ to accommodate (A-11). To simplify notation, I suppress subscript $i$.

Let $\{\mathcal{F}_t\}_{0 \leq t \leq 1}$ denote the filtration generated by the path of observed performance $Z(\cdot)$. Suppose that, given a contract $s(Z^1)$, an $\mathcal{F}_\tau$-adapted process $\epsilon$ solves problem (A-11) with certain equivalent $\chi$. Let

$$
F(\tau; \epsilon'; m) \equiv E_m \left[ \exp \left( s(Z^1) - \int_0^\tau c(\epsilon'(t))dt - \int_0^1 c(m(t))dt \right) \right]_{| \mathcal{F}_\tau} 
$$

$$
= F(\tau; \epsilon; m)K(\tau; \epsilon'), 
$$

where

$$
K(\tau; \epsilon') \equiv \exp \left( \int_0^\tau \left[ k(\epsilon(t)) - k(\epsilon'(t)) \right] dt \right). 
$$

$F$ is the conditional expected utility at time $\tau$ if the worker has followed an effort sequence $\epsilon'$ for tasks $[0, \tau]$ and then switches to a sequence $m$ for the remainder of the production process. Let $V(\tau; \epsilon')$ be the maximal value of the worker’s problem given the information at time $\tau$ if the worker has followed an effort sequence $\epsilon'$ for tasks $[0, \tau]$. Then,

$$
V(\tau; \epsilon') \equiv \max_m F(\tau; \epsilon'; m) 
$$

$$
= \max_m F(\tau; \epsilon; m)C(\tau; \epsilon') = V(\tau; \epsilon)C(\tau; \epsilon'). 
$$

(A-12)

Since $V(\tau; \epsilon) = F(\tau; \epsilon; \epsilon)$, the law of iterated expectations implies $E_x[V(\tau'; \epsilon) | \mathcal{F}_\tau] = V(\tau; \epsilon)$ for $\tau' > \tau \geq 0$. Therefore, $V(\tau; \epsilon)$ is a martingale relative to $\{\mathcal{F}_\tau\}_{0 \leq \tau \leq 1}$. Since $\epsilon$ is $\mathcal{F}_\tau$-adapted, $V(\tau; \epsilon)$ is also a martingale relative to the filtration generated by the driftless
Brownian motion $Z(\tau) - \int_0^\tau \mu(\epsilon(t)) \, dt$. By the martingale representation theorem (e.g. Øksendal (2003), ch. 4), there exists a unique, square-integrable and $\mathcal{F}_\tau$-measurable stochastic process $\gamma \equiv \{\gamma(t); t \in [0, 1]\}$ such that

$$dV(\tau; \epsilon) = \gamma(\tau) \left[ Z(\tau) - \int_0^\tau \mu(\epsilon(t)) \, dt \right]. \quad (A-13)$$

For any effort sequence $\epsilon'$, $dZ = \mu(\epsilon') + dB$; thus $dV(\tau; \epsilon) = \gamma \left[ \mu(\epsilon') - \mu(\epsilon) \right] dt + \gamma dB$. Together with (A-12), this implies

$$dV(\tau; \epsilon') = d[V(\tau; \epsilon) K(\tau; \epsilon')] = \{ \gamma \left[ \mu(\epsilon') - \mu(\epsilon) \right] + [k(\epsilon) - k(\epsilon')] V(\tau; \epsilon) \} K(\tau; \epsilon') \, dt + \gamma K(\tau; \epsilon') dB.$$

By the Principle of Optimality, if $\epsilon'$ is an optimal effort sequence, then it maximizes the drift of $V(\tau; \epsilon')$. By hypothesis, $\epsilon$ is optimal for the worker; thus it satisfies the following first-order necessary condition:

$$\gamma(t) \mu'(\epsilon(t)) = k'(\epsilon(t)) V(t; \epsilon), \quad (A-14)$$

for all $t \in [0, 1]$.

Let $\chi(t)$ denote the certain equivalent corresponding to $V(t; \epsilon)$. Therefore, $\chi(t)$ satisfies

$$E \left[ \chi(t) \exp \left( - \int_0^1 c(\epsilon(t')) \, dt' \right) | \mathcal{F}_t \right] = V(t; \epsilon).$$

Solving for $\chi(t)$ yields

$$\chi(t) = \frac{V(t; \epsilon)}{E \left[ \exp \left( - \int_0^1 k(\epsilon(t')) \, dt' \right) | \mathcal{F}_t \right].} \quad (A-15)$$

Since the time derivative of the denominator in (A-15) is zero,

$$\frac{d\chi(t)}{\chi(t)} = \frac{dV(t; \epsilon)}{V(t; \epsilon)} = \frac{\gamma(t)}{V(t; \epsilon)} \left[ Z(t) - \int_0^t \mu(\epsilon(t')) \, dt' \right] = \frac{k'(\epsilon(t))}{\mu'(\epsilon(t))} \left[ Z(t) - \int_0^t \mu(\epsilon(t')) \, dt' \right], \quad (A-16)$$

where the second and third lines follow from (A-13) and (A-14), respectively. Using Ito’s Lemma for the function $\ln(\cdot)$ yields

$$d\ln \chi(t) = \frac{d\chi(t)}{\chi(t)} - \frac{1}{2} \frac{1}{[\chi(t)]^2} \left[ d\chi(t) \right]^2 = \frac{d\chi(t)}{\chi(t)} - \frac{1}{2} \frac{1}{[\chi(t)]^2} \left[ \chi(t) \frac{k'(\epsilon(t))}{\mu'(\epsilon(t))} \right]^2 dt = \frac{d\chi(t)}{\chi(t)} - \frac{1}{2} \left[ \frac{k'(\epsilon(t))}{\mu'(\epsilon(t))} \right]^2 dt. \quad (A-17)$$
Using (A-16) to substitute for \(d\chi(t)/\chi(t)\), integrating (A-17) and letting \(\chi(0) = \chi\) yields
\[
\ln \chi(1) = \ln \chi + \int_0^1 \frac{k'(\epsilon(t))}{\mu'(\epsilon(t))}[dZ(t) - \mu(\epsilon(t))dt] - \frac{1}{2} \int_0^1 \left[\frac{k'(\epsilon(t))}{\mu'(\epsilon(t))}\right]^2 dt.
\] (A-18)

By the construction of \(\chi(t)\), \(\chi(1) = \exp[s(Z^1)]\); thus, \(\chi(1) = w(Z^1)\). Substituting the latter in (A-18) delivers (A-10).

**Step 2.** Following the ‘first-order’ approach in the principal-agent literature (Schaettler and Sung (1993)), I formulate and solve the firm’s relaxed optimal contracting problem, in which the incentive compatibility constraints in Problem (5) are replaced with the contract representations obtained in step 1. Importantly, the solution to this problem is not necessarily implementable by the contracts (A-10), since the latter were derived from only necessary conditions for optimality in the worker’s problem. This issue is tackled in Step 3.

To obtain the firm’s relaxed problem, insert (A-10) in the objective function of Problem (5) and drop the incentive compatibility constraints:
\[
\min_{\{\chi_i, \epsilon; i \in [0, h]\}} \int_0^h \chi_i \mathbb{E}\left[\exp\left(\int_0^1 \frac{k'(\epsilon_i(t))}{\mu'(\epsilon_i(t))}[dZ_i(t) - \mu(\epsilon_i(t))dt] - \frac{1}{2} \int_0^1 \left[\frac{k'(\epsilon_i(t))}{\mu'(\epsilon_i(t))}\right]^2 dt\right)\right] di
\]

\[\text{s.t.} \quad (i) \quad n_0 = -h^{-1} \int_0^h \mathbb{E}[Z_i(1)] \, di\]
\[ (ii) \quad Z_i(t) = \int_0^t \mu(\epsilon_i(t')) \, dt' + B_i(t), \text{ for } i \in [0, h] \]
\[ (iii) \quad \mathbb{E}[U(w_i, \epsilon_i)] \geq \bar{\pi}, \text{ for } i \in [0, h] \] (A-19)

The following steps simplify this problem. First, substitute \(\mathbb{E}[Z_i(1)] = \int_0^1 \mathbb{E}[\mu(\epsilon_i(t))] \, dt\) in (i). Second, substitute (ii) in the objective function and use the fact that
\[
\mathbb{E}\left[\exp\left(\int_0^1 \frac{k'(\epsilon_i(t))}{\mu'(\epsilon_i(t))}dB_i(t) - \frac{1}{2} \int_0^1 \left[\frac{k'(\epsilon_i(t))}{\mu'(\epsilon_i(t))}\right]^2 dt\right)\right] = 1,
\]
for any \(\epsilon_i\).\(^{47}\) Third, individual rationality constraints should bind at the optimum.\(^{48}\) Expression (iii) can then be used to solve for \(\chi_i\) as a function of \(\epsilon_i\),
\[
\chi_i = \frac{\bar{\pi}}{\mathbb{E}\left[\exp\left(-\int_0^1 k(\epsilon_i) dt\right)\right]}.
\] (A-20)

Problem (A-19) is then simplified to a problem of finding the optimal \(\epsilon_i\) that minimizes the certainty equivalent \(\chi_i\) for \(i \in [0, h]\), subject to a single performance constraint:
\[
\min_{\{\epsilon; i \in [0, h]\}} \int_0^h \frac{\bar{\pi}}{\mathbb{E}\left[\exp\left(-\int_0^1 k(\epsilon) dt\right)\right]} di,
\]

\[\text{s.t.} \quad n_0 = -h^{-1} \int_0^h \int_0^1 \mathbb{E}[\mu(\epsilon_i(t))] \, dt \, di.\] (A-21)

\(^{47}\)Under the assumption that \(k(\cdot)\) and \(\mu(\cdot)\) have continuous derivatives, then \(k'(\cdot)/\mu'(\cdot)\) is bounded for all \(\epsilon_i(t) \in [\epsilon_L, \epsilon_H]\). Then the process \(\exp\left(\int_0^t \frac{k'(\epsilon_i(t))}{\mu'(\epsilon_i(t))}dB_i(t) - \frac{1}{2} \int_0^t \left[\frac{k'(\epsilon_i(t))}{\mu'(\epsilon_i(t))}\right]^2 dt\right)\) is an \(\mathcal{F}_T\)-martingale with expected value equal to one, for all \(\tau\) and \(\epsilon\). See Karatzas and Shreve (1988), p.200.

\(^{48}\)Suppose that IR constraints didn’t bind for a positive measure of workers. Then it would be possible to decrease the certainty equivalent of these workers, shifting down their corresponding wage functions while holding the effort sequences constant.
Note that both the objective and constraint of problem (A-21) are independent of $B_i(t)$ for all $i \in [0, h]$ and $t \in [0, 1]$. This implies that, without loss of generality, the domain for admissible effort sequences in (A-21) can be restricted to the set of deterministic (history-independent) sequences.

Dropping the expectations operator and disregarding the (positive) constant $\pi$, (A-21) can be further simplified into

$$\min_{\{\epsilon_i; i \in [0, h]\}} \int_0^h \exp \left[ \int_0^1 k(\epsilon_i(t))dt \right] \, di \quad \text{s.t.} \quad n_0 \geq -h^{-1} \int_0^h \int_0^1 \mu(\epsilon_i(t)) \, dt \, di. \quad \text{(A-22)}$$

It is convenient to analyze this problem by introducing a set of auxiliary choice variables $\{a_i \in (-\mu(\epsilon_H), -\mu(\epsilon_L)); i \in [0, h]\}$, satisfying $n_0 = h^{-1} \int_0^h a_i \, di$, interpreted as an allocation of $n_0 h$ mistakes across $h$ workers. I compute the solution to (A-22) sequentially with the following two-step procedure:

1. For given $\{a_i \in (-\mu(\epsilon_H), -\mu(\epsilon_L)); i \in [0, h]\}$, determine the effort sequence $\epsilon_i$ that solves, for each $i$,

$$\min_{\epsilon_i} \int_0^1 k(\epsilon_i(t)) \, dt \quad \text{s.t.} \quad a_i \geq \int_0^1 -\mu(\epsilon_i(t)) \, dt. \quad \text{(A-23)}$$

Under the assumptions that $k(\cdot)$ and $\mu(\cdot)$ are convex and strictly concave, respectively, it is straightforward to verify that the solution to (A-23) is a unique constant effort for all $t \in [0, 1]$, denoted $\epsilon(a_i)$, that satisfies $a_i = -\mu(\epsilon(a_i))$. In addition, $a_i \in (-\mu(\epsilon_H), -\mu(\epsilon_L))$ implies $\epsilon(a_i) \in (\epsilon_L, \epsilon_H]$.

2. Given $\epsilon(a_i)$, determine the optimal allocation of mistakes across workers that solves

$$\min_{\{a_i \in [-\mu(\epsilon_H), -\mu(\epsilon_L)]; i \in [0, h]\}} \int_0^h \exp [k(\epsilon(a_i))] \, di \quad \text{s.t.} \quad n_0 = h^{-1} \int_0^h a_i \, di. \quad \text{(A-24)}$$

Since $\exp [k(\cdot)]$ is strictly convex, the solution to (A-24) is a unique constant allocation of mistakes across workers satisfying the constraint of the problem; that is, $a_i = n_0$ for all $i \in [0, h]$.

In light of these results, I conclude that there exists a unique solution to problem (A-19), in which every worker exerts an identical constant effort throughout the production process. This solution, denoted $\epsilon^*$, satisfies $n_0 = -\mu(\epsilon^*)$ for all $i \in [0, h]$ and $t \in [0, 1]$. In addition, $n_0 \in (-\mu(\epsilon_H), -\mu(\epsilon_L))$ implies $\epsilon^* \in (\epsilon_L, \epsilon_H]$.

**Step 3.** I check the validity of the first-order approach by verifying that the solution to Problem (A-19) is implementable. That is, I show that if worker $i$ is assigned contract (A-10) evaluated at effort $\epsilon^*$, then a constant effort $\epsilon^*$ is the a.s. unique maximizer of worker $i$’s expected utility. This step is needed because the wage representations in step 1 were derived from only necessary conditions for optimality in the worker’s problem.\(^{49}\)

Evaluating the wage representation (A-10) at a constant effort $\epsilon^*$ and using the expression for the certainty equivalent (A-20) at $t = 1$, yields\(^{50}\)

\(^{49}\)Schaettler and Sung (1993) provide an in-depth analysis of the first-order approach to the moral hazard problem in a continuous-time environment.

\(^{50}\)From (A-20), the certainty equivalent for a constant effort $\epsilon^*$ is $\chi_i = \pi/E \left[ \exp \left( -\int_0^1 k(\epsilon_i) \, dt \right) \right] = \pi/\exp (-k(\epsilon^*))$. Therefore, $\ln \chi_i = \ln \pi + k(\epsilon^*)$. 

33
\[
\ln w_i(Z_t^1) = \ln u + k(e^*) + \frac{k'(e^*)}{\mu'(e^*)}Z_t(1) - \frac{k'(e^*)}{\mu'(e^*)}\mu(e^*) - \frac{1}{2} \left[ \frac{k'(e^*)}{\mu'(e^*)} \right]^2.
\]

Define constants \( \alpha^* \) and \( \beta^* \) such that,

\[
\alpha^* \equiv \ln u + k(e^*) - \frac{k'(e^*)}{\mu'(e^*)}\mu(e^*) - \frac{1}{2} \left[ \frac{k'(e^*)}{\mu'(e^*)} \right]^2,
\]

\[
\beta^* \equiv \frac{k'(e^*)}{\mu'(e^*)}.
\]

The worker’s problem becomes,

\[
\max_{\epsilon} E \left[ \exp (\alpha^* + \beta^* Z_t(1)) - \int_0^1 k(\epsilon(t))dt \right] \quad s.t \quad Z_t(1) = \int_0^1 \mu(\epsilon(t))dt + B_t(1), \quad (A-25)
\]

where \( \epsilon \) is an adapted process. Substituting the constraint in the objective function and rearranging yields,

\[
E \left[ \exp (\alpha^* + \beta^* Z_t(1)) - \int_0^1 k(\epsilon(t))dt \right] = \exp(\alpha^*)E \left[ \exp \left( \int_0^1 [\beta^* \mu(\epsilon(t)) - k(\epsilon(t))] dt + \beta^* B_t(1) \right) \right].
\]

The distribution of \( B_t(1) \) is independent of \( \epsilon(t) \), for any \( t \in [0,1] \). This implies that, for any realization of \( B_t(1) \), utility is maximized if and only if \( \epsilon \) maximizes

\[
J(\epsilon) \equiv \int_0^1 [\beta^* \mu(\epsilon(t)) - k(\epsilon(t))] dt.
\]

Any effort strategy that mandates the worker to deviate from maximizing \( J \) will reduce expected utility. Therefore, optimal effort is deterministic. Clearly, \( J \) is maximized when effort in task \( t, \epsilon(t) \), maximizes \( \beta^* \mu(\epsilon(t)) - k(\epsilon(t)) \) for all \( t \in [0,1] \). The convexity and strict concavity of \( k(\cdot) \) and \( \mu(\cdot) \), respectively, imply that there is an a.s. unique effort, denoted \( \hat{\epsilon} \), which is constant for all \( t \in [0,1] \) and solves the worker’s problem \((A-25)\). In particular, \( \hat{\epsilon} \) satisfies

\[
\beta^* = \frac{k'(\hat{\epsilon})}{\mu'(\hat{\epsilon})} = \frac{k'(e^*)}{\mu'(e^*)},
\]

where the second equality follows by the definition of \( \beta^* \).

Under Assumption (1), \( k'(\cdot)/\mu'(\cdot) \) is a strictly increasing function. It follows that \( \hat{\epsilon} = e^* \), and therefore \( e^* \) is a.s. uniquely implemented by contract \( \ln w_i = \alpha^* + \beta^* Z_t(1) \).

### A.3 Proof of Corollary 1

From Proposition 1, the (stochastic) wage of worker \( i \) is

\[
w_i^* = \underline{w}\exp[k(e^*) + \beta(\epsilon)[B_t(1) - \frac{1}{2}\beta(\epsilon)]], \quad (A-26)
\]
for all \( i \in [0, h] \), where \( \beta(\epsilon) = k'(\epsilon)/\mu'(\epsilon) \) and \( \epsilon_L < \epsilon \leq \epsilon_H \). Because \( B_i(1) \) is normally distributed with mean zero and unit variance, it follows that \( E[\exp(\beta(\epsilon)B_i(1))|\epsilon] = \exp[\beta(\epsilon)^2/2] \). Therefore, \( E[w_i^*|\epsilon] = \pi \exp[k(\epsilon)] \), as stated in part (a) of Corollary 1.

For part (b), consider two effort levels \( \epsilon_1 \) and \( \epsilon_2 \) such that \( \epsilon_L < \epsilon_1 < \epsilon_2 \leq \epsilon_H \). From Proposition 1,

\[
\text{Var} \left( \log(w_i^*) \right) \left| \epsilon \right. = \beta(\epsilon)^2,
\]

since \( Z_i(1) \) is normally distributed with mean \( \mu(\epsilon) \) and unit variance. Under Assumption (1), \( 0 < \beta(\epsilon_1) < \beta(\epsilon_2) \), which establishes (i).

To prove statement (ii) of part (b), first rescale firm-level wages by their mean to obtain normalized wages. If the firm implements effort \( \epsilon_j \), \( j = \{1, 2\} \), the normalized wages can be written as

\[
\hat{w}_{ij}(x) \equiv \frac{w_{ij}^*}{E[w_i^*|\epsilon_j]} = e^{\beta(\epsilon_j)[B_i(1) - \frac{1}{2}\beta(\epsilon_j)]},
\]

where the second equality follows from (A-26) and part (a) of Corollary 1. Normalized wages are log-normally distributed; in particular,

\[
\ln(\hat{w}_{ij}(x)) \overset{d}{\sim} N\left(-\frac{1}{2}\beta(\epsilon_j)^2, \beta(\epsilon_j)^2\right).
\]

Observe that: (a) \( E[\ln(\hat{w}_{ij}(x)) | \epsilon_j] \) is strictly decreasing in \( \epsilon_j \); (b) \( \text{Var}[\ln(\hat{w}_{ij}(x)) | \epsilon_j] \) is strictly increasing in \( \epsilon_j \); (c) \( E[\ln(\hat{w}_{ij}(x)) | \epsilon_j] = -\text{Var}[\ln(\hat{w}_{ij}(x)) | \epsilon_j]/2 \), for \( j = \{1, 2\} \). Conditions (a), (b) and (c) are sufficient to conclude that the firm-level distribution of normalized wages when the firm implements effort \( \epsilon_1 \) second-order stochastically dominates the firm-level distribution of normalized wages when the firm implements effort \( \epsilon_2 \) (see Levy (1973), Theorem 5). This completes the proof of (ii).

### A.4 Profit Maximization

#### A.4.1 Optimality of \( q(\theta) \)

In this section, I show that the expression for firm quality (16) is optimal in problem (7); i.e., \( q(\theta) = q_\mu(\theta) = q_\tau(\theta) \), under the functional form assumptions (13)-(15). By equation (9), this is equivalent to showing that \( q(\theta) \) is the unique global minimizer of the average cost of quality \( c^\theta(q)/q \) for \( q \in [q_L, q_H] \).

Recall that \( c^\theta(q) \equiv \omega(\epsilon(\theta, q))/\theta^s \), where \( \epsilon(\theta, q) \) implicitly defined by \( q = q(\theta, -\mu(\epsilon)) \). Under (13)-(15),

\[
c^\theta(q) = \pi \exp[k(\epsilon(\theta, q))] \theta^{-s}, = \pi \exp \left[ k \left( \frac{\log q}{(\gamma \log \theta)^2} \right)^{1/(1-\gamma)} \right] \theta^{-s}, = \pi \exp \left[ \lambda \left( \log q \right)^{1/(1-\gamma)} \right] \theta^{-s}, \tag{A-27}
\]

\[\text{If } \epsilon = \epsilon_L, \text{ Corollary 1 holds trivially. In this case, } \beta(\epsilon_L) = 0 \text{ and thus } w_i^* = \pi \exp[k(\epsilon_L)] \text{ for all } i. \text{ Note also that } \beta(\epsilon) > 0 \text{ and thus inequality is strictly positive whenever } \epsilon_L < \epsilon \leq \epsilon_H. \text{ The rest of the proof focuses on the case } \epsilon_L < \epsilon \leq \epsilon_H. \]
where $\Lambda \equiv k/(\gamma \log \theta)^{z/(1-z)}$ is independent of $q$. For any $q > 0$,
\[
\frac{d \left( c^\theta(q)/q \right)}{dq} = \pi \theta^{-s} \exp \left[ \Lambda (\log q)^{1/(1-z)} \right] q^{-2} \left( \frac{\Lambda}{1-z} (\log q)^{z/(1-z)} - 1 \right).
\]  
(A-28)

Moreover, $\pi \theta^{-s} \exp \left[ \Lambda (\log q)^{1/(1-z)} \right] q^{-2} > 0$ for all $\theta \in [\theta_L, \theta_H]$ and $q \in [q_L, q_H]$.

From (16),
\[
[\log (q(\theta))]^{z/(1-z)} = \left[ \frac{1-z}{k} \right] \left[ \gamma \log(\theta) \right]^{z/(1-z)} = \frac{1-z}{\Lambda}.
\]  
(A-29)

For any $q > 0$, (A-28) and (A-29) imply
\[
\frac{d \left( c^\theta(q)/q \right)}{dq} = \begin{cases} 
< 0 & \text{if } q < q(\theta), \\
= 0 & \text{if } q = q(\theta), \\
> 0 & \text{if } q > q(\theta).
\end{cases}
\]

If $q(\theta) \in [q_L, q_H]$, then $q(\theta)$ is the unique global minimizer of the average cost of quality $c^\theta(q)/q$ and therefore optimal in problem (7). As shown in the text, $q(\theta) \in [q_L, q_H]$ provided that effort is defined over a sufficiently large interval.

### A.4.2 Equilibrium Unit Costs and Prices Across Firms

This section analyses the variation in equilibrium unit costs and output prices across firms. Let $c^\theta_s \equiv c^\theta(q(\theta))$ denote the equilibrium unit cost in firm $\theta$. Then,
\[
c^\theta_s = \pi \exp \left[ k \left( \frac{\log q(\theta)}{(\gamma \log \theta)^z} \right)^{1/(1-z)} \right] \theta^{-s},
\]
\[
= \pi \exp \left[ \gamma k \left( \frac{1-z}{k} \right)^{1/z} \log(\theta) \right] \theta^{-s},
\]
\[
= \pi \theta^\gamma \left[ (1-z)^{1/z} k^{(1-z)/z} \right]^{-s},
\]
where the first and second lines follow from (A-27) and (16), respectively. Equilibrium unit costs increase across firms if and only if $\gamma \left[ (1-z)^{1/z} k^{(1-z)/z} \right] > s$. This pattern reflects two countervailing forces. First, optimal quality increases in $\theta$. Higher $\gamma$ and $k$ imply a higher elasticity of quality with respect to productivity and a higher cost of effort, respectively. Second, if $s > 0$ then labor productivity increases in $\theta$. Higher $s$ implies that the firm requires fewer workers to produce one unit of output.

With CES demand, output prices are constant mark-ups over marginal costs. Therefore, for sufficiently high $\gamma$ and $k$ or small $s$, the model delivers positive correlations between output prices, average wages, employment and revenue across firms, which is consistent with the empirical evidence documented in Kugler and Verhoogen (2012).

### A.5 Proof of Proposition 2

Coming soon...

Since $\lim_{\theta_m \to 0} J(\theta_m) = \infty$ and $\lim_{\theta_m \to \infty} J(\theta_m) = 0$, $\theta_L < \theta_d$ is satisfied by a sufficiently small $\theta_L$, for finite trade costs.
A.6 Proof of Lemma 2

Preliminaries. From equation (27), re-write the employment distribution across firms in equilibrium \( j \in \{0, 1\} \) as

\[
G_{h,j}(\theta) = \begin{cases} 
0, & \theta_L \leq \theta \leq \theta_{d,j}, \\
D_j^{-1} \int_{\theta_{d,j}}^{\theta} (\theta')^{\kappa-1} dG_{\theta}(\theta'), & \theta_{d,j} \leq \theta \leq \theta_{x,j}, \\
D_j^{-1} \int_{\theta_{d,j}}^{\theta} \Upsilon_j(\theta')(\theta')^{\kappa-1} dG_{\theta}(\theta'), & \theta_{x,j} \leq \theta \leq \theta_H,
\end{cases}
\]  

(A-30)

where

\[
D_j \equiv \int_{\theta_{d,j}}^{\theta} \Upsilon_j(\theta')(\theta')^{\kappa-1} dG_{\theta}(\theta'),
\]

\[
\Upsilon_j(\theta') = \begin{cases} 
\Upsilon_{x,j} & \text{if } \theta' \geq \theta_{x,j}, \\
1 & \text{if } \theta' < \theta_{x,j},
\end{cases}
\]  

(A-31)

Letting \( \theta_{x,0} \rightarrow \theta_H \) or \( \theta_{x,1} \rightarrow \theta_{d,1} \) in (A-30) yields the employment distribution corresponding to autarky or to the equilibrium in which all firms export following trade liberalization, respectively.

To determine how cutoffs change in response to trade liberalization, first note that since \( \nu_0 > \nu_1 \) by hypothesis, then \( \theta_{d,1}/\theta_{x,1} > \theta_{d,0}/\theta_{x,0} \) by equation (23) and thus \( \Upsilon_{x,1} > \Upsilon_{x,0} \). In addition, the free entry condition (24) implies that \( \theta_{d,0} < \theta_{d,1} \) if and only if \( \theta_{x,0} > \theta_{x,1} \); i.e., cutoffs change in opposite directions as a result of trade liberalization. Given \( \nu_0 > \nu_1 \), \( \nu_1 \in [L, \widehat{\tau}] \), and assuming that productivity is defined over a sufficiently large interval (see Section 4), then \( \theta_L < \theta_{d,0} < \theta_{d,1} \leq \theta_{x,1} < \theta_{x,0} \) where \( \theta_{d,1} = \theta_{x,1} \) if all firms export following trade liberalization. Moreover, \( \nu_0 \geq \widehat{\tau} \) if and only if \( \theta_{x,0} \geq \theta_H \). As a result, the productivity cutoffs in equilibria before and after trade liberalization satisfy either: (i) \( \theta_L < \theta_{d,0} < \theta_{d,1} \leq \theta_{x,1} < \theta_{x,0} < \theta_H \) if \( \nu_0 < \tau \) or (ii) \( \theta_L < \theta_{d,0} < \theta_{d,1} \leq \theta_{x,1} < \theta_{x,0} \) if \( \nu_0 > \tau \).

For the analysis below, it is convenient to partition the domain of \( \theta \) into regions denoted by \( R_i, i \in \{A, B, C, D, E\} \). When the initial equilibrium is not autarkic, \( \nu_0 < \tau \), let \( R_A = [\theta_L, \theta_{d,0}] \), \( R_B = [\theta_{d,0}, \theta_{d,1}] \), \( R_C = [\theta_{d,1}, \theta_{x,1}] \), \( R_D = [\theta_{x,1}, \theta_{x,0}] \), \( R_E = [\theta_{x,0}, \theta_H] \). Otherwise, if \( \nu_0 \geq \tau \), let \( R_A = [\theta_L, \theta_{d,0}] \), \( R_B = [\theta_{d,0}, \theta_{d,1}] \), \( R_C = [\theta_{d,1}, \theta_{x,1}] \), \( R_D = [\theta_{x,1}, \theta_{x,0}] \), \( R_E = [\theta_{x,0}, \theta_H] \). In either case, if all firms export in \( j = 1 \), then \( \theta_{x,1} = \theta_{d,1} \) and \( RC = \{\theta_{d,1}\} \).

The following preliminary result introduces three properties of \( G_{h,j} \), denoted P1, P2 and P3, that are used repeatedly in the proof of Lemma 2.

Lemma A-1 Let \( (s_C, s_D, s_E) = (\Lambda, \Lambda/\Upsilon_{x,1}, \Lambda\Upsilon_{x,0}/\Upsilon_{x,1}) \), where \( \Lambda \equiv D_1/D_0 \) is positive and independent of \( \theta \).

P1. For \( j \in \{0, 1\} \), \( G_{h,j}(\theta) \) is non-decreasing, continuous and piecewise differentiable in \([\theta_L, \theta_H]\). Moreover, for \( i \in \{C, D, E\} \), \( G'_{h,j}(\theta) > 0 \) for all \( \theta \) in the interior of \( R_i \).

P2. If \( \theta^* \in R_i \) and \( \theta^{**} \in R_i \), \( i \in \{C, D, E\} \), then

\[
G_{h,1}(\theta^*) - G_{h,0}(\theta^*) = G_{h,1}(\theta^{**}) - G_{h,0}(\theta^{**}) + (1 - s_i) \int_{\theta^{**}}^{\theta^*} G'_{h,1}(v) dv.
\]  

(A-32)
**P3.** Suppose that \( G_{h,0} \) and \( G_{h,1} \) intersect at a point \( \tilde{\theta} \) in \( R_i \), for \( i = \{C, D, E\} \). If \( R_i \) has a non-empty interior, then:

(i) \( G_{h,1} \) intersects \( G_{h,0} \) once in \( R_i \), from below, if and only if \( s_i < 1 \).

(ii) \( G_{h,1} \) intersects \( G_{h,0} \) once in \( R_i \), from above, if and only if \( s_i > 1 \).

(iii) \( G_{h,0}(\theta) = G_{h,1}(\theta) \) for all \( \theta \) in \( R_i \) if and only if \( s_i = 1 \).

**Proof.** P1 is immediately verified from (A-30). To establish P2, first note that the slopes of \( G_{h,0} \) and \( G_{h,1} \) satisfy the following ‘proportionality property’ in the interior of regions C, D and E:

\[
G_{h,0}'(\theta) = \begin{cases} 
\frac{\Delta}{\mathcal{Y}_{x,1}} G_{h,1}'(\theta), & \text{if } \theta \in R_C, \\
\frac{1}{\mathcal{Y}_{x,1}} G_{h,1}'(\theta), & \text{if } \theta \in R_D, \\
\frac{\Delta}{\mathcal{Y}_{x,1}} G_{h,1}'(\theta), & \text{if } \theta \in R_E.
\end{cases}
\]

(A-33)

Next, fix a region \( R_i, i = \{C, D, E\} \), and consider \( \theta^* \in R_i \) and \( \theta^{**} \in R_i \). Then,

\[
G_{h,1}(\theta^*) - G_{h,0}(\theta^*) = G_{h,1}(\theta^{**}) + [G_{h,1}(\theta^*) - G_{h,1}(\theta^{**})] - G_{h,0}(\theta^{**}) - [G_{h,0}(\theta^*) - G_{h,0}(\theta^{**})],
\]

\[
= G_{h,1}(\theta^{**}) - G_{h,0}(\theta^{**}) + \int_{\theta^{**}}^{\theta^*} [G_{h,1}'(v) - G_{h,0}'(v)] dv,
\]

\[
= G_{h,1}(\theta^{**}) - G_{h,0}(\theta^{**}) + (1 - s_i) \int_{\theta^{**}}^{\theta^*} G_{h,1}'(v) dv,
\]

where the last line uses (A-33).

P3 is a corollary of P2. Fix a region \( i, i = \{C, D, E\} \). For any \( \theta \in R_i, \theta \neq \tilde{\theta} \), setting \( \theta = \theta^* \) and \( \tilde{\theta} = \theta^{**} \) in expression (A-32) yields

\[
G_{h,1}(\theta) - G_{h,0}(\theta) = (1 - s_i) \int_{\theta}^{\tilde{\theta}} G_{h,1}'(v) dv.
\]

(A-34)

By P1, \( G_{h,1}' > 0 \) in the interior of \( R_i \). Therefore, \( s_i < 1 \) if and only if \( G_{h,1}(\theta) < G_{h,0}(\theta) \) for \( \theta < \tilde{\theta} \) and \( G_{h,1}(\theta) > G_{h,0}(\theta) \) for \( \theta > \tilde{\theta} \). In this case, \( G_{h,1}(\theta) \) intersects \( G_{h,0}(\theta) \) once, from below at point \( \tilde{\theta} \).\(^{52}\) A similar argument establishes the single-crossing property from above if and only if \( s_i > 1 \). Finally, setting \( s_i = 1 \) in (A-34) yields \( G_{h,1}(\theta) = G_{h,0}(\theta) \) for all \( \theta \in R_i \). For the converse, if \( G_{h,1}(\theta) = G_{h,0}(\theta) \) for some \( \theta \in R_i \), then P1 and (A-34) imply \( s_i = 1 \). \( \blacksquare \)

**Part (a) of Lemma 2.** If \( \nu_0 \geq \nu \), the initial equilibrium is autarkic. I show that \( G_{h,1}(\theta) \leq G_{h,0}(\theta) \) in each region of the domain of \( \theta \) (i.e. \( R_i, i = \{A, B, C, D\} \)), with strict inequality for some \( \theta \in [\theta_L, \theta_H] \).

For region D, note that \( G_{h,1} \) intersects \( G_{h,0} \) at point \( \theta_H \), where \( \theta_H \in R_D \). Suppose that \( G_{h,1} \) intersects \( G_{h,0} \) from above at point \( \theta_H \). Then \( \mathcal{Y}_{x,1} > 1 \) and (P3) imply \( s_C > s_D > 1 \). In addition, by (P3) the intersection is unique in \( R_D \), thus \( G_{h,1}(\theta_{x,1}) > G_{h,0}(\theta_{x,1}) \) by continuity

\(^{52}\)Technically, this argument applies to any \( \theta \) in the interior of \( R_i \). However, the continuity of \( G_{h,j} \) ensures that the conclusion can be extended to the boundary of \( R_i \).
(P1). Since $\theta_{d,1} \in R_C$ and $\theta_{x,1} \in R_C$, let $\theta^* = \theta_{d,1}$ and $\theta^{**} = \theta_{x,1}$ in P2, which implies $G_{h,1}(\theta_{d,1}) > 0$, which is false. Then, $G_{h,1}$ must intersect $G_{h,0}$ from below at point $\theta_H$. By (P3) the intersection is unique in $R_D$, thus $G_{h,1}(\theta) \leq G_{h,0}(\theta)$ for all $\theta \in R_D$, with equality if and only if $\theta = \theta_H$. For region $C$, $\theta_{x,1} \in R_D \cap R_C$ implies $G_{h,1}(\theta_{x,1}) < G_{h,0}(\theta_{x,1})$. Since $G_{h,1}(\theta_{d,1}) = 0 < G_{h,0}(\theta_{d,1})$, continuity (P1) ensures that $G_{h,1}$ and $G_{h,0}$ do not intersect in the interior of region $C$. Therefore, $G_{h,1}(\theta) < G_{h,0}(\theta)$ for all $\theta \in R_C$. For $\theta \in R_B$, $G_{h,0}(\theta) \geq 0 = G_{h,1}(\theta)$, with strict inequality if $\theta > \theta_{d,0}$. Finally, for $\theta \in R_A$, $G_{h,0}(\theta) = 0 = G_{h,1}(\theta)$.

Part (b) of Lemma 2.

**Case:** $G_{h,1}(\hat{\theta}) \leq G_{h,0}(\hat{\theta})$ for some $\hat{\theta} \in [\theta_{x,0}, \theta_H)$. I show that $G_{h,1}(\theta) \leq G_{h,0}(\theta)$ in each region of the domain of $\theta$ (i.e. $R_i$, $i = \{A, B, C, D, E\}$), with strict inequality for some $\theta \in [\theta_L, \theta_H]$.

First, since $\hat{\theta} \in R_E$, then $G_{h,1}(\hat{\theta}) \leq G_{h,0}(\hat{\theta})$ implies that $G_{h,1}$ intersects $G_{h,0}$ from below at point $\theta_H$. By P3, $s_E \leq 1$ and $G_{h,1}(\theta) \leq G_{h,0}(\theta)$ for all $\theta \in R_E$. For region $D$, $s_E \leq 1$ and $\forall \theta_{x,0} > 1$ imply $s_D < 1$. Note that $\theta_{x,0} \in R_D \cap R_E$ and $\theta_{x,0} \geq 0$ for all $\theta \in R_D$. Therefore, letting $\theta^{**} = \theta_{x,0}$ in P2 yields $G_{h,1}(\theta) < G_{h,0}(\theta)$ for $\theta < \theta_{x,0}$ for all $\theta \in R_D$. For region $C$, suppose that $G_{h,1}$ intersects $G_{h,0}$ at point $\theta$ in $R_C$. Since $\theta_{x,1} \in R_C \cap R_D$ and $G_{h,1}(\theta_{x,1}) < G_{h,0}(\theta_{x,1})$, P3 implies that the intersection is from above and unique. Moreover, $s_C > 1$. Letting $\theta^* = \theta_{d,1}$ in P2,

$$G_{h,1}(\theta_{d,1}) - G_{h,0}(\theta_{d,1}) = G_{h,1}(\theta) - G_{h,0}(\theta) + (1 - s_C) \int_{\theta_{d,1}}^{\theta_{d,0}} G_{h,0}'(v)dv \geq 0,$$

with strict inequality if $\theta_{d,1} < \theta$. But then $G_{h,0}(\theta_{d,1}) > 0$ implies $G_{h,1}(\theta_{d,1}) > 0$, which is false. Therefore, $G_{h,1}$ does not intersect $G_{h,0}$ in $R_C$. Since both employment functions are continuous by P1, $G_{h,1}(\theta_{d,1}) = 0 < G_{h,0}(\theta_{d,1})$ and $G_{h,1}(\theta_{x,1}) < G_{h,0}(\theta_{x,1})$ imply $G_{h,1}(\theta) < G_{h,0}(\theta)$ for all $\theta \in R_C$. For $\theta \in R_B$, $G_{h,0}(\theta) \geq 0 = G_{h,1}(\theta)$, with strict inequality if $\theta > \theta_{d,0}$. Finally, for $\theta \in R_A$, $G_{h,0}(\theta) = 0 = G_{h,1}(\theta)$.

**Case:** $G_{h,1}(\hat{\theta}) > G_{h,0}(\hat{\theta})$ for some $\hat{\theta} \in [\theta_{x,0}, \theta_H)$. I show that $G_{h,1}$ intersects $G_{h,0}$ once, from below, in regions $C$, $D$ and interior of $E$.

Since $\hat{\theta} \in R_E$, then $G_{h,1}$ intersects $G_{h,0}$ from above at point $\theta_H$. By P3, $s_E > 1$ and $G_{h,1}(\theta) > G_{h,0}(\theta)$ for all $\theta$ in the interior of region $E$. Next, suppose that $G_{h,1}$ does not intersect $G_{h,0}$ in region $D$. By P1, both employment distributions are continuous and thus $G_{h,1}(\theta_{x,0}) > G_{h,0}(\theta_{x,0})$ implies $G_{h,1}(\theta_{x,1}) > G_{h,0}(\theta_{x,1})$. Letting $\theta^* = \theta_{d,1}$ in P2,

$$G_{h,1}(\theta_{d,1}) - G_{h,0}(\theta_{d,1}) = G_{h,1}(\theta_{x,1}) - G_{h,0}(\theta_{x,1}) + (1 - s_D) \int_{\theta_{d,1}}^{\theta_{d,0}} G_{h,1}'(v)dv > 0,$$

since $s_D > s_E > 1$. But then $G_{h,1}(\theta_{d,1}) > 0$, which is false. Therefore $G_{h,1}$ intersects $G_{h,0}$ in region $D$. Moreover, by P3 the intersection is unique and, because $G_{h,1}(\theta_{x,0}) > G_{h,0}(\theta_{x,0})$, from below. This implies that, in region $C$, $G_{h,1}(\theta_{x,1}) \leq G_{h,0}(\theta_{x,1})$, with equality if and only if $G_{h,1}$ intersects $G_{h,0}$ at point $\theta_{x,1}$. Since $G_{h,1}(\theta_{d,1}) = 0 < G_{h,0}(\theta_{d,1})$, continuity (P1) ensures that $G_{h,1}$ and $G_{h,0}$ do not intersect in the interior of region $C$. 

39
A.7 Proof of Proposition 3

In light of Lemma 2, it is sufficient to show that Assumption (2) implies \( G_{h,1}(\hat{\theta}) \leq G_{h,0}(\hat{\theta}) \) for some \( \hat{\theta} \in [\theta_{x,0}, \theta_H] \), when \( \iota_0 < \iota \). From the employment distribution (27),

\[
1 - G_{h,j}(\hat{\theta}) = \frac{\int_{\theta_0}^{\hat{\theta}} \theta^\Gamma \theta^{\kappa c} dG_{\theta}(\theta)}{(Y_{x,j})^{-1} \int_{\theta_d}^{\theta_0} \theta^\Gamma \theta^{\kappa c} dG_{\theta}(\theta) + \int_{\theta_x}^{\theta_0} \theta^\Gamma \theta^{\kappa c} dG_{\theta}(\theta)},
\]

(A-35)

for \( j \in \{0, 1\} \), where \( (Y_{x,j})^{-1} = 1/(1 + (\iota_j)^{-\rho/(1-\rho)}) \in (0,1) \). Therefore, \( G_{h,1}(\hat{\theta}) \leq G_{h,0}(\hat{\theta}) \) if and only if the denominator of (A-35) is increasing in the variable trade cost. Without loss of generality, I focus on infinitesimal changes in \( \iota \) and show that \( D'(\iota) > 0 \), where

\[
D(\iota) = (Y_x)^{-1} \int_{\theta_d}^{\theta_x} \theta^\Gamma \theta^{\kappa c} dG_{\theta}(\theta) + \int_{\theta_x}^{\theta_0} \theta^\Gamma \theta^{\kappa c} dG_{\theta}(\theta),
\]

after dropping index \( j \) to simplify notation. Note that \( Y_x, \theta_d \) and \( \theta_x \) are functions of \( \iota \). Therefore,

\[
D'(\iota) = \frac{\rho}{1 - \rho} (Y_x)^{-2} \Gamma^{-1/(1-\rho)} \int_{\theta_d}^{\theta_x} \theta^\Gamma \theta^{\kappa c} dG_{\theta}(\theta) + \frac{\rho}{1 - \rho} (Y_x)^{-2} \Gamma^{-1/(1-\rho)} \[ (Y_x)^{-1} - 1 \] (\theta_x)^{\Gamma-\kappa c} \theta'_x(\iota) g_\theta(\theta_x) - (Y_x)^{-1} (\theta_d)^{\Gamma-\kappa c} \theta'_d(\iota) g_\theta(\theta_d),
\]

(A-36)

where \( \theta'_m(\iota) \equiv d\theta_m/d\iota \) for \( m \in \{d, x\} \). The last line follows from \( (Y_x)^{-1} - 1 = -(Y_x)^{-1} \Gamma - \rho/(1-\rho) \) (by definition of \( Y_x \)) and the inequality:

\[
\Gamma^{-1/(1-\rho)} \left( \frac{\theta_x}{\theta_d} \right)^{\Gamma-\kappa c} < \Gamma^{-1/(1-\rho)} \left( \frac{\theta_x}{\theta_d} \right)^{\Gamma} = \frac{f_x}{f_d},
\]

which uses \( \theta_x > \theta_d > 0, \Gamma > k\kappa_c > 0 \) and (23).

Recall that, in a symmetric equilibrium, the expression for relative cutoffs (23), the free entry condition (24) and the monotonicity of \( J(\cdot) \) imply \( \theta'_d(\iota) < 0 \) and \( \theta'_x(\iota) > 0 \). Therefore, from (A-36), \( D'(\iota) > 0 \) if

\[
\frac{f_x g_\theta(\theta_x)}{f_d g_\theta(\theta_d)} < -\frac{\theta'_d(\iota)}{\theta'_x(\iota)} = \frac{f_x J'(\theta_x)}{f_d J'(\theta_d)},
\]

where the equality follows by applying the Implicit Function Theorem on the free entry condition (24). To conclude, \( D'(\iota) > 0 \) if

\[
\frac{J'(\theta_d)}{g_\theta(\theta_d)} < \frac{J'(\theta_x)}{g_\theta(\theta_x)},
\]

which, given \( \theta_x > \theta_d > 0 \), is guaranteed by Assumption (2).
A.8 Productivity Distributions that Satisfy Assumption (2)

Recall that $J (\theta) \equiv \int_0^{\theta_H} \left( (v/\theta)^\Gamma - 1 \right) dG(v)$. Therefore,

$$J'(\theta) = -\Gamma \theta^{-(\Gamma+1)} \int_0^{\theta_H} \theta^\Gamma v^\Gamma dG(v),$$  \hspace{1cm} (A-37)

and

$$J''(\theta) = - (\Gamma + 1) \frac{J'(\theta)}{\theta} + \Gamma \frac{g_\theta(\theta)}{\theta}. \hspace{1cm} (A-38)$$

Densities with Elasticity Greater Than Or Equal To $-(\Gamma + 1)$. From (A-38), it follows that

$$d \frac{d}{d\theta} \left( \frac{J'(\theta)}{g_\theta(\theta)} \right) \geq 0 \iff J''(\theta) g_\theta(\theta) \geq J'(\theta) g'_\theta(\theta),$$

$$\iff -J'(\theta) \left[ (\Gamma + 1) \frac{1}{\theta} + \frac{g'_\theta(\theta)}{g_\theta(\theta)} \right] \geq -\Gamma \frac{g_\theta(\theta)}{\theta}. \hspace{1cm} (A-39)$$

Next, let $\varepsilon(\theta) \equiv \theta g'_\theta(\theta)/g_\theta(\theta) \theta$ denote the elasticity of $g_\theta(\theta)$ at point $\theta \in [\theta_L, \theta_H]$. Since $J'(\theta) < 0$, equation (A-39) implies

$$\varepsilon(\theta) \geq - (\Gamma + 1) \Rightarrow d \frac{d}{d\theta} \left( \frac{J'(\theta)}{g_\theta(\theta)} \right) \geq 0.$$

Therefore, Assumption (2) is satisfied by a class of productivity densities satisfying $\varepsilon(\theta) \geq - (\Gamma + 1)$ for all $\theta \in [\theta_L, \theta_H]$. Distributions with non-decreasing densities satisfy $\varepsilon(\theta) \geq 0$ for all $\theta \in [\theta_L, \theta_H]$, thus they are included in this class.

Truncated Pareto Distribution. Consider $g_\theta(\theta) = z (\theta_L)^z \theta^{-z-1}/[1 - (\theta_L/\theta_H)^z]$, $z > 0$, for $\theta \in [\theta_L, \theta_H]$. From equation (A-37),

$$\frac{J'(\theta)}{g_\theta(\theta)} = \frac{\Gamma}{z - \Gamma} [\theta_L^z \Gamma \theta_H^{-z} - 1].$$

Therefore,

$$d \frac{d}{d\theta} \left( \frac{J'(\theta)}{g_\theta(\theta)} \right) = \Gamma \theta_H^{z-\Gamma} \theta_L^{-z-\Gamma-1} > 0,$$

for all $z > 0$. For a truncated Pareto distribution, $\varepsilon(\theta) = -(z+1)$ for all $\theta \in [\theta_L, \theta_H]$. Therefore, not every truncated Pareto belongs to the class of productivity densities satisfying $\varepsilon(\theta) \geq - (\Gamma + 1)$ for all $\theta \in [\theta_L, \theta_H]$. Still, all of them satisfy Assumption (2).

A.9 Proof of Proposition 4

From part (a) of Corollary (1) and the expression for optimal effort (17), $\omega(\theta) = \bar{u}_j \theta^{k\kappa_c}$. From expression (28), this implies

$$dG_{w,j}(\theta) = \frac{\theta^{k\kappa_c}}{\int_{\theta_H}^{\theta_H} v^{k\kappa_c} dG_{h,j}(v)} dG_{h,j}(\theta),$$
for all $\theta \in [\theta_{d,j}, \theta_{H}]$. Next, let $\tilde{\Lambda} \equiv \Lambda \tilde{D}_1/\tilde{D}_0 > 0$, where $\tilde{D}_j \equiv \int_{\theta_{d,j}}^{\theta_{H}} v^{k\kappa} dG_{h,j}(v)$ for $j \in \{0, 1\}$ and $\Lambda$ is defined as in Lemma (A-1). Therefore,

$$G'_{w,0}(\theta) = \begin{cases} 
\tilde{\Lambda}G'_{w,0}(\theta), & \text{if } \theta \in R_C, \\
\frac{\Lambda}{Y_{x,1}} G'_{w,0}(\theta), & \text{if } \theta \in R_D, \\
\frac{\tilde{\Lambda}Y_{x,0}}{Y_{x,1}} G'_{w,0}(\theta), & \text{if } \theta \in R_E.
\end{cases}$$

where $R_i$ is defined as in the proof of Lemma (2), for $i \in \{C, D, E\}$.

The slopes of $G_{w,0}$ and $G_{w,1}$ thus satisfy a proportionality property which is identical to the proportionality property for employment distributions, after a redefinition of the positive constant $\Lambda$, in expression (A-33). Since the exact definition of $\Lambda$ is immaterial in the proof of Lemma (A-1), then $G_{w,0}$ and $G_{w,1}$ satisfy P1, P2 and P3, after replacing $\Lambda$ with $\tilde{\Lambda}$. It is then trivial to adjust the proof of Lemma (2) to show:

(a) If $t_0 \geq \tau$, then $G_{w,1}$ first-order stochastically dominates $G_{w,0}$.
(b) If $t_0 < \tau$, consider $\hat{\theta} \in [\theta_{x,0}, \theta_{H}]$:

- If $G_{w,1}(\hat{\theta}) \leq G_{w,0}(\hat{\theta})$, then $G_{w,1}$ first-order stochastically dominates $G_{w,0}$.
- If $G_{w,1}(\hat{\theta}) > G_{w,0}(\hat{\theta})$, then $G_{w,1}$ intersects $G_{w,0}$ once, from below, in $[\theta_{d,1}, \theta_{H}]$.

To establish Proposition (4), it thus suffices to show that, if $t_0 < \tau$, then Assumption (2) implies $G_{w,1}(\hat{\theta}) \leq G_{w,0}(\hat{\theta})$, for any $\hat{\theta} \in [\theta_{x,0}, \theta_{H})$.

Suppose that $t_0 < \tau$ and fix $\hat{\theta} \in [\theta_{x,0}, \theta_{H})$. Using the cross-firm wage distribution (28) together with (i) $\omega(\theta) = \pi_j \theta^{k\kappa}$, (ii) $dG_{h,j}(\theta) = M_j L^{-1} h_j(\theta) dG_{h,j}(\theta)$, (iii) $h_j(\theta) = \kappa_y Y_j(\theta) (A_j \pi_j^{-1})^{1/(1-\rho)} \theta^{r-k\kappa}$ and (iv) definition of $Y_j(\theta)$ in equation (A-31), yields

$$1 - G_{w,j}(\hat{\theta}) = \frac{\int_{\theta_{d,j}}^{\theta_{H}} \theta^r dG_{\theta,j}(\theta)}{(Y_{x,j})^{-1} \int_{\theta_{d,j}}^{\theta_{H}} \theta^r dG_{\theta,j}(\theta) + \int_{\theta_{x,j}}^{\theta_{H}} \theta^r dG_{\theta,j}(\theta)},$$

(A-40)

for $j \in \{0, 1\}$, where $(Y_{x,j})^{-1} = 1/(1 + (t_j)^{-\rho/(1-\rho)}) \in (0, 1)$. Therefore, $G_{w,1}(\hat{\theta}) \leq G_{w,0}(\hat{\theta})$ if and only if the denominator of (A-40) is increasing in the variable trade cost. I proceed by adjusting the proof of Proposition 3 to show that $D'(\tau) > 0$, where

$$D(\tau) \equiv (Y_{x})^{-1} \int_{\theta_d}^{\theta_x} \theta^r dG_{\theta}(\theta) + \int_{\theta_x}^{\theta_H} \theta^r dG_{\theta}(\theta),$$

and dropping index $j$ to simplify notation. Following the steps leading to equation (A-36) yields

$$D'(\tau) > (Y_{x})^{-1} (\theta_d)^r \left\{ - \frac{f_x}{f_d} \theta_x'(\tau) g_\theta(\theta_x) - \theta_x'(\tau) g_\theta(\theta_d) \right\}.$$

(A-41)

As shown in the proof of Proposition 3, Assumption (2) guarantees that the right-hand side of (A-41) is positive. Therefore, Assumption (2) implies $G_{w,1}(\hat{\theta}) \leq G_{w,0}(\hat{\theta})$ for any $\hat{\theta} \in [\theta_{x,0}, \theta_{H})$, which completes the proof.

42