Equilibria and Efficiency in Bilingual Labour Markets

Alex Armstrong*

August 28, 2013

Abstract

We consider a labour market where two languages are commonly in use and each individual may make a costly investment to learn the language which is not his or her mother tongue. Language skills are productive in a human capital sense and can also be used to signal unobservable traits to employers. Due to the informational asymmetry between workers and employees, the equilibrium rate of bilingualism in the economy may exceed the socially efficient level. On the other hand, the spillovers associated with second language acquisition may imply there is not enough bilingualism relative to the social optimum. We consider the circumstances under which either the "signalling" or the "network" welfare effects dominate in equilibrium. Depending on the parameter values of the model, policies of either encouraging or discouraging the investment in language skills may be welfare enhancing.

Keywords: Language Skill Returns, Linguistic Equilibria. Language Policy. JEL Codes: D62, D83, J24.

*Queen’s University Department of Economics, email: armstronga@econ.queensu.ca.
1 Introduction

A fundamental source of inefficiency in competitive markets is the presence of asymmetric information. The inability of economic agents to verify the quality of goods subject to trade can lead to violations of the first welfare theorem even when those possessing superior information can take costly actions to signal quality. It is well known that the same kind of failure can occur in labour markets as demonstrated by Spence (1973)’s model of educational signalling. In this case, there are instances where educational investments are individually rational but socially unproductive.

In principle, the same argument applies to investments in the knowledge of a second language. Language skills are costly to acquire and may serve as a verifiable signal to employers that an individual possesses desirable attributes as an employee. However acquired language skills and more general forms of education differ in important ways. First, the ability to learn a second language is a particularly noisy signal of unobservable productivity. While there is a correlation between linguistic abilities and favorable cognitive abilities and personality traits, the correlation is weak. The efficiency with which an individual is able to learn a second language depends primarily on exposure to the language, which typically conveys little information about the innate productive potential of the individual. Second, languages are technologies that depend fundamentally on complementarities. The value of knowing any particular language depends on the extent to which it is understood by others. An implication of this is that the acquisition of a second language may indirectly benefit the speakers of that language. Consequently there may be a production externality associated with second language acquisition that is not considered by individuals who make a rational cost-benefit analysis regarding their investment decision.

This paper formalizes these observations by presenting a model of a labour market where wages depend both on the worker’s innate ability in production and on his language skills. It is assumed that two languages coexist in the economy so that a worker who chooses to learn a second language is rewarded with a bilingual wage premium which is determined endogenously by the equilibrium distribution of language skills. In general, the equilibrium is socially inefficient due to two counter-
acting effects. First, the decision to become bilingual is associated with a positive network welfare effect since learning a second language indirectly contributes to the productivity of monoglot speakers of the acquired language. Second, since ability in second language learning and unobservable abilities are assumed to be only weakly correlated, pure separating equilibria do not occur. This contrasts with standard job market signalling models where workers’ ability types can be fully revealed by their investment decisions. Here, the acquisition of a second language alters the wage distribution through a signalling effect which is shown to decrease welfare at the margin. At the equilibrium, social welfare may be improved by either increasing or decreasing the rates of bilingual knowledge in each of the communities.

There is a substantial body of empirical literature concerning the relationship between earnings and language skills in a variety of contexts.\(^1\) One strand of this research concerns the returns to bilingualism in labour markets where two or more languages are commonly in use. Most but not all of this work has been done using Canadian Census data. Examples include Shapiro and Stelcner (1997), Albouy (2008), Christofides and Swidinsky (2010) and Nadeau (2010).\(^2\) In general, it has been concluded that wage premia are enjoyed by those individuals who can speak more than one language, although the size of the premium depends on both the worker’s first language and the prevalence of the minority language speakers in the economy. The standard methodology is to estimate log-linear wage regressions that include dummy variables indicating the mother tongue and second language skills of the worker. In explaining the earnings differences between language groups, authors have made reference to both human capital theory and signalling theory but there has been little done in the way of formal modelling to explain the demand for language skills. Two exceptions are Carliner (1981) and Bloom and Grenier (1992) who sketch out arguments suggesting that the demand for language skills depends on the underlying

---

\(^1\)See especially the work of Barry Chiswick and Paul Miller, a good selection of which has been republished in the volume Chiswick and Miller (2007).

linguistic demography of the population in question. Indeed, this idea is implicitly adopted by most of these empirical studies since it has been standard practise to estimate separate regressions for Quebec and Canada outside of Quebec and, in some cases, for Anglophones and Francophones.

Related is a theoretical literature on second language acquisition initiated by Selton and Pool (1991) and Church and King (1993) and elaborated more recently by Ginsburgh et al. (2007), Gabszewicz et al (2011a) and Gabszewicz et al (2011b). This literature interprets second language acquisition in terms of a noncooperative game where the payoffs are a generic benefit with no specific reference to the labour market. The present paper is most closely related to Church and King (1993) and Gabszewicz et al (2011a) both of which analyze the equilibrium distribution of language skills in a bilingual economy. Church and King (1993) model a simple economy where individual utility is increasing in the proportion of the population with whom one shares a language. They assume that the costs of second language acquisition are constant across individuals so that multiple corner solutions - where either all or none of a particular group becomes bilingual - are possible. If the costs of learning are low enough it is socially efficient for the minority language group to become bilingual. However it is possible in equilibrium that either no one becomes bilingual or only the majority language group becomes bilingual. The authors conclude that a policy of subsidizing learning of the majority language may be called for in these circumstances. Gabszewicz et al (2011a) extend Church and King’s model to include heterogenous learning cost distributions which allows for the possibility of interior equilibrium solutions. They also show that, for a range of parameter values, the equilibrium levels of second language learning are below the social welfare maximizing levels.

This paper contributes to the theoretical literature on second language acquisition in the following ways. First, we relate the acquisition decision directly to the market return to language skills: something that is readily observable in the data. Second,

---

3 Also see Lazear (1999) for a model of language learning in a similar vein.
4 Robinson (1988) develops a model of second language acquisition in the context of the labour market although the bilingual wage premium is exogenous.
the acquisition decision in the model here is partially motivated by a signalling benefit which has not been considered in previous theoretical models. Third, the model results lead to novel implications concerning the welfare effects of language policies.

In order to motivate the theoretical analysis presented below, table 1 presents estimates of the bilingual wage premium along with statistics related to the linguistic demography of Canada, Quebec and the Rest of Canada (R.O.C.) based on an analysis of the 2006 Canadian Census Public Use Microdata File (Individuals). The dataset consists of the responses of 2.7% of the Canadian population to the long form census which includes questions on earnings, employment, education and demographics as well as on language use and knowledge. The sample is limited to include only Francophones and Anglophones who were between 17 and 65 years of age, who were not self-employed, worked full-time and earned more than $1000 in wage income in 2005. The dependent variable is the log of wage and salary income and the controls used are: age, age squared, marital status dummy variables, place of residence dummy variables, and highest educational degree dummy variables. Two sets of estimates are presented, one that includes controls for occupation and industry and one that does not.

Francophones are a minority in Canada overall as well as outside of the province of Quebec, while Anglophones are a minority within Quebec. As may be expected, rates of bilingual knowledge are higher among the minority language group, since being able to speak the language of the majority enables access to social and economic resources. Indeed, the benefit of this access is reflected in the wage premia. There is a statistically significant premium paid to bilingual Francophones both inside and outside of Quebec. Bilingual Anglophones also earn a premium, although it is much smaller outside of Quebec and in some cases not statistically significant. This suggests that the wage is an increasing function of what Gabszewicz et al (2011b) refer to as one’s *communicative benefit* from learning a second language: the increase in the proportion of the population with whom one shares a language. For example, for a Francophone living in a region where 50% of the population is Francophone, 50%

5The full regression results are reported in the appendix - tables 2.A.1 and 2.A.2.
Table 1: The Distribution of Language Skills and Bilingual Wage Premia in Canada 2005

<table>
<thead>
<tr>
<th></th>
<th>% Who Can Speak</th>
<th>Bilingual Wage Premium (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>% of Population</td>
<td>Both Official Languages</td>
</tr>
<tr>
<td></td>
<td>Males</td>
<td>Females</td>
</tr>
<tr>
<td>Canada</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Anglophones</td>
<td>58.0</td>
<td>9.4</td>
</tr>
<tr>
<td>Francophones</td>
<td>22.1</td>
<td>42.4</td>
</tr>
<tr>
<td>Quebec</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Anglophones</td>
<td>7.8</td>
<td>69.0</td>
</tr>
<tr>
<td>Francophones</td>
<td>80.1</td>
<td>35.8</td>
</tr>
<tr>
<td>Rest of Canada</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Anglophones</td>
<td>73.6</td>
<td>7.4</td>
</tr>
<tr>
<td>Francophones</td>
<td>3.4</td>
<td>83.6</td>
</tr>
</tbody>
</table>

Data Source: 2006 Canadian Census. Full regression results available upon request. ** p < .01. * p < .05.

is Anglophone and 50% of the Anglophones can speak French, the communicative benefit to learning English is 25%: the proportion of Anglophones who don’t speak French.

In the case of Anglophones living outside of Quebec, the communicative benefit is small since the majority is Anglophone and a large proportion of the Francophone minority is bilingual. Within Quebec, the communicative benefit is relatively large as approximately half of the population is unable to speak English. This line of argument also explains the significant bilingual wage premium for Francophones outside of Quebec where the communicative benefit is very large. For Francophones within Quebec, the communicative benefit is much smaller, however there is still a significant bilingual wage premium. It seems likely that this is due to the status of cultural and economic dominance that English enjoys, not just in Quebec, but in North America and around the world.

In line with the preceding argument, the following section develops a model of second language acquisition and bilingual wage premium formation. Subsection 2.1 presents the full information human capital version of the model. The following subsection 2.2 develops an alternative model version where language skills are not
productive but second language acquisition serves as a signal to employers of unobservable ability. Subsection 2.3 synthesizes the human capital and signalling models. Section 3 concludes with a brief discussion of policy implications.

2 Model

We consider a population of individuals, each of whom is endowed with one of two mother tongues: language $x$ and language $y$. The mother tongue is a language the individual grew up speaking, which cost them nothing to learn and which they will never lose. In what follows we refer to the two subsets of individuals with either language as a mother tongue as the language communities $x$ and $y$. Normalizing the measure or size of the total population to unity, the size of language community $x$ is given by $\chi$ and the size of community $y$ is $1 - \chi$ where $\chi \in (0, 1)$.

Beyond membership in one of the language communities, individuals are differentiated in two other respects: by their abilities in second language learning and by their non-linguistic productive capacities on the labour market. Language learning ability is captured by the cost term $c$ which is a continuous random variable with support $[0, 1]$ and non-linguistic productive capacity is captured by a binary parameter $\theta \in \{l, h\}$ where $l$ and $h$ refer to low and high ability respectively ($l < h$). The cost term $c$ is taken to be a reduced form representing both individual aptitude in second language acquisition and any exogenous circumstances that otherwise affect the ability of the individual to become bilingual. The proportion of high types in each community is $\mu \in (0, 1)$ which is the same for each language community. The probability that an individual in language community $j = x, y$ with ability type $\theta$ has a learning cost below $c$ is given by the conditional cumulative distribution function $G_j(c|\theta)$. We impose the following assumptions:

A.1. $G_j(c|\theta)$ is continuous and strictly increasing.

A.2. $G_j(c|l) < G_j(c|h)$ for $j = x, y$ and all $c \in (0, \infty)$

A.3. $G_y(c|\theta) \leq G_x(c|\theta)$ for $\theta = l, h$ and all $c \in (0, \infty)$.
Assumptions A.1 is imposed to minimize technical complications in what follows. Assumptions A.2 and A.3 deserve some motivation. According to assumption A.2, \( G_j(c|l) \) first-order stochastically dominates \( G_j(c|h) \) which implies that there is a weak correlation between ability in second language acquisition and nonlinguistic abilities. The literature on second language acquisition highlights both cognitive abilities and personality traits as important determinants of individual variation in linguistic aptitude (Dörnyei (2006)). Key among these cognitive abilities is *working memory* (Miyake and Friedman (1998)) which involves the "temporary storage and manipulation of information that is assumed to be necessary for a wide range of complex and cognitive activities" (Baddeley (2003): p. 189). Personality traits that have been found to be influential in second language acquisition include internal motivation and the degree of extroversion. So to the extent that these personal attributes also contribute to productivity in a broad sense, it is reasonable to assume that language learning abilities and productivity unrelated to linguistic skills are at least weakly correlated.\(^6\)

According to assumption A.3, the cost distributions for community \( y \) (weakly) stochastically dominate those for community \( x \). This allows for the possibility that one of the languages - in this case \( y \) - is, on average, easier to learn than the other. Such a situation may arise for purely structural linguistic reasons as when one language is phonetically or syntactically more complex than the other. Alternatively, one of the languages may enjoy a status of cultural dominance. In that case, the non-dominant community is exposed to the dominant language through media channels - music, television, internet and so forth - that diminish the costs of becoming bilingual.

Utility is assumed to be separable in income and the cost of language learning. Define the indicator variable \( \delta \) where \( \delta = 1 \) if the individual learns the other language

---

\(^6\)Many bilingual individuals learn their second language in childhood due to the active involvement of their parents. Recognizing this, the decision-making unit in the model may be broadened to refer to the family rather than just the individual, so long as the parents are understood to be acting in the interests of the child. Those children who grow up in bilingual households - where each parent speaks a different language - have a learning cost parameter value that lies close to zero.
and $\delta = 0$ if she does not.\(^7\) Then utility is given by:

$$u(\theta, \delta, c) = w_j(\theta, \delta) - \delta c$$

(1)

where $w_j(\theta, \delta)$ is the wage paid to type $\theta$ individual in community $j$ conditional on $\delta$. The only decision made by the individual is whether or not to become bilingual.\(^8\) We assume that she will become bilingual in case of indifference, so $\delta(\theta, \delta) = 1$ when:

$$w_j(\theta, 1) - c \geq w_j(\theta, 0)$$

(2)

Evaluating (2) as an equality defines a cutoff value $c^* (\theta) = w_j(\theta, 1) - w_j(\theta, 0)$ for the cost term, below which the individual becomes bilingual and above which she does not. In what follows we refer to $\Delta w_{\theta j} \equiv w_j(\theta, 1) - w_j(\theta, 0)$ as the bilingual wage premium for type $\theta = l, h$ in language community $j = x, y$. For given wage premia, the proportions of each ability type who become bilingual in community $j$ are given by:

$$b_{\theta j} = G_j(\Delta w_{\theta j}|\theta), \quad \theta = l, h, \quad j = x, y$$

(3)

and the total proportion of the population who becomes bilingual in community $j$ is:

$$B_j = \mu b_{lj} + (1 - \mu) b_{hj}$$

(4)

Although individuals take the wage premium as given when making their decision, it is determined in equilibrium by the rates of language learning by each community and by the behaviour of firms on the demand side of the labour market.

The productivity of an individual in a job is a function of her non-linguistic ability $\theta$ and of $p$, the proportion of the population with whom she shares an language. We

\(^7\)In the interests of clarity, the analysis neglects the issue of variable fluency in the second language. Either the individual becomes bilingual or does not. If variable fluency were to be included, the solution concept would refer to a continuous distribution of language skills rather than a binary distribution. In either case, the main conclusions of the paper would remain the same.

\(^8\)It should be noted that in this specification, there is no inherent value to learning a second language beyond the earnings advantage it entails.
assume the functional form:

\[ f(p, \theta) = p^\alpha \theta \]  

(5)

where \( \alpha \in [0, 1] \) is the elasticity of language skill complementarity in production. In case \( \alpha = 0 \), language skills are completely unproductive.\(^9\) We would expect \( \alpha \) to vary with the structure of the economy. For example, a shift towards services and away from manufacturing would tend to increase \( \alpha \). Additionally, greater urbanization would be associated with a higher value of \( \alpha \).

Since there are only two languages, for all bilingual individuals, \( p = 1 \). The value of \( p \) for monoglots depends on the rate of second language learning by the other language community. So when the proportion \( B_y \) of language community \( y \) is bilingual, for monoglots in community \( x \) : \( p = \chi + (1 - \chi) B_y \). Likewise for monoglots in community \( y \) : \( p = (1 - \chi) + \chi B_y \). For ease of exposition we introduce the functions:

\[
m_x(\chi, B_y) = \chi + (1 - \chi) B_y \\
\]

(6)

\[
m_y(\chi, B_x) = (1 - \chi) + \chi B_x
\]

2.1 Pure Human Capital Model

As a baseline, we first assume that firms can observe both the language skills of workers and their non-linguistic abilities. In the context of wage formation, it is irrelevant whether language learning costs are observable to employers since they play no direct role in productivity. A set of firms makes wage offers à la Bertrand so that the Nash equilibrium entails workers being paid wages equal to their productivity.\(^10\) Without loss of generality, we characterize wage formation for language community \( x \). For bilingual individuals of ability type \( \theta \), \( w(\theta, 1) = \theta \) and for monoglots of ability

\(^9\)The assumption that language skills and non-linguistic abilities are complements in production is restrictive but not unreasonable. Consider as an extreme example the value of a genius who is unable to communicate with his manager or coworkers.

\(^{10}\)Workers of high and low type are perfect substitutes in production.
type $\theta$, $w(\theta, 0) = m_x^\alpha \theta$. So for community $x$ there are two bilingual wage premia:

$$\Delta w_{yx} = (1 - m_x^\alpha) \theta, \quad \theta = l, h$$  \hspace{1cm} (7)

Since $h > l$ by assumption, it is clear that $\Delta w_{hx} > \Delta w_{lx}$. Furthermore, due to assumption A.3, $b_{hx} > b_{lx}$: the proportion of bilingual high types will always be strictly greater than the proportion of bilingual low types.

For both ability types, wage premia and, by extension, the rates of language learning in community $x$ depend negatively on $B_y$, the rate of language learning in community $y$. In particular, for a given $B_y$ the total rate of bilingual learning in community $x$ is determined by the reaction function $B_x(B_y) = \mu b_{hx} + (1 - \mu) b_{lx}$ where $b_{hx}$ and $b_{lx}$ are functions of $B_y$ through (3) and (7):

$$B_x(B_y) = \mu G_x((1 - [m_x(\chi, B_y)]^\alpha) h|h) + (1 - \mu) G_x((1 - [m_x(\chi, B_y)]^\alpha l)|l)$$ \hspace{1cm} (8)

For language community $y$ the counterpart expression to (8) is:

$$B_y(B_x) = \mu G_y((1 - [m_y(\chi, B_x)]^\alpha h)|h) + (1 - \mu) G_y((1 - [m_y(\chi, B_x)]^\alpha l)|l)$$ \hspace{1cm} (9)

We refer to any solution to the system of equations (8) and (9), as an equilibrium distribution of language skills.

Figure 1 graphically depicts one such equilibrium as the intersection of the reaction curves $B_x(B_y)$ and $B_y(B_x)$ in $\{B_y, B_x\}$ space. Depending on the shape of the functions $G_j(c|\theta)$ multiple equilibria are possible. The consideration of multiple equilibria does not substantially alter the analysis beyond adding some tedious complications. To avoid this possibility, we impose the following sufficient condition:
A.4. Let:

\[ E [g_x (c|\theta)] = \mu g_x (c|h) + (1 - \mu) g_x (c|l), \]
\[ E [g_y (c|\theta)] = \mu g_y (c|h) + (1 - \mu) g_y (c|l) \]

where \( g_j (c|\theta) \equiv G^j \) denotes the conditional probability density function for \( j = x, y \). Then for all \( c \in [0, \infty) \):

\[ E [g_x (c|\theta)] E [g_y (c|\theta)] < 1 \]

This leads to the following:

**Proposition 1** Under assumption A.4, the solution \( \{ B^*_x, B^*_y \} \) to the system (8) and (9) is the unique stable equilibrium distribution of language skills in the pure human capital model.

We leave all proofs for the appendix. At this level of generality, there is no closed form solution for the equilibrium distribution of language skills. However, it
is relatively straightforward to show how the equilibrium depends on the relative sizes of the language communities:

**Proposition 2** Let \( B^*_x(\chi) \) and \( B^*_y(\chi) \) denote the equilibrium distribution of language skills as functions of \( \chi \). Then:

1. The equilibrium rates of second language acquisition for each community decrease with the size of the community:
   \[
   \frac{dB^*_x}{d\chi} < 0, \quad \frac{dB^*_y}{d\chi} > 0
   \]

2. \( B^*_x \) goes to zero as \( \chi \) approaches one and \( B^*_y \) goes to zero as \( \chi \) approaches zero.

The content of proposition 2 is intuitively appealing: the benefit of learning a second language, as reflected in the bilingual wage premium, is increasing in the proportion of the population who have that language as a mother tongue.

The equilibrium distribution of language skills is associated with equilibrium wage premium for each community and ability type. In order to characterize how the premia depend on the relative sizes of the language communities and the cost distribution functions, define:

\[
\Gamma \equiv \left[ \mu \int_0^\infty cdG_x(c|h) + (1-u) \int_0^\infty cdG_x(c|l) \right] - \left[ \mu \int_0^\infty cdG_y(c|h) + (1-u) \int_0^\infty cdG_y(c|l) \right]
\] (10)

Since \( G_y(c|\theta) \leq G_x(c|\theta), \Gamma \geq 0 \). Furthermore \( \Gamma = 0 \) only if \( G_y(c|\theta) = G_x(c|\theta) \) for \( \theta = l, h \). So the value \( \Gamma \) is a summary measure of how much more difficult \( x \) is to learn than \( y \). Initially suppose that \( \chi = \frac{1}{2} \) and \( G_x(c|\theta) = G_y(c|\theta), \theta = l, h \). The symmetry of the reaction curves imply that \( B^*_x = B^*_y \) and by implication: \( \Delta w_{\theta x} = \Delta w_{\theta y}, \) the wage premia are the same for both communities. Now consider the effect of perturbing the cost distributions so that \( \Gamma > 0 \). Language \( x \) becomes relatively more difficult to learn so that fewer individuals in community \( y \) become bilingual.
This will increase the wage premium for community $x$ which in turn encourages more individuals in that community to become bilingual. This drives down the wage premium in community $y$ so that when $\Gamma > 0$ results in $\Delta w_{yx} > \Delta w_{y\theta y}$.

Now suppose again that $\chi = \frac{1}{2}$ and $G_x(c|\theta) = G_y(G(c|\theta), \theta = l, h$, and consider a perturbation of $\chi$ holding the cost distributions constant. When $\chi > \frac{1}{2}$, proposition 1 implies that $B_y^* > B_x^*$, so $\Delta w_{yx} < \Delta w_{y\theta y}$. Conversely, $\chi < \frac{1}{2}$ implies that $B_y^* < B_x^*$, so $\Delta w_{yx} > \Delta w_{y\theta y}$. These results are summarized in the form of a phase diagram in $\{\Gamma, \chi\}$ space in figure 2. Due to the continuity of the cost distributions and definitions of the wage premia, there is a locus of values of $\Gamma$ and $\chi$ such that $\Delta w_{yx} = \Delta w_{y\theta y}$.

Since language learning by one community increases the productivity of monoglots in the other community, the private and social benefits of second language acquisition at the equilibrium fail to coincide. To demonstrate this, define the social welfare of $\theta$-types in community $x$ as the per capita unweighted sum of utilities for a particular cutoff learning cost value $c^*$ where those with $c \leq c^*$ are bilingual and those with
$c > c^*$ are monoglots.\(^{11}\) Since $G(c|\theta)$ is strictly increasing we can invert the definition $b_{\theta x} = G(c^*|\theta)$ to express social welfare in terms of $b_{\theta x}$ and $m_x$:

$$s_{\theta x}(b_{\theta x}, m_x) \equiv \int_0^{G_x^{-1}(b_{\theta x})} [\theta - c] \, dG_x(c|\theta) + [1 - b_{\theta x}] \, m_x^\alpha \theta \quad (11)$$

where the first term on the right-hand-side is the summed utilities of bilingual individuals and the second term is the summed utilities of monoglots. Then define the social welfare of the entire population as:

$$SW(b_{hx}, b_{lx}, b_{hy}, b_{ly}) = \chi (\mu s_{hx} + (1 - \mu) s_{lx}) + (1 - \chi) (\mu s_{hy} + (1 - \mu) s_{ly}) \quad (12)$$

Note that, due to the linearity of utility, there is no consideration of equity in this social welfare specification. In what follows we refer to $\partial SW/\partial b_{hj}$ as the net social benefit of the marginal learner of ability type $\theta$ in community $j = x, y$. So for high types in community $x$, the net benefit is\(^{12}\):

$$\frac{\partial SW}{\partial b_{hx}} = \chi \mu \left( h - G_x^{-1}(b_{hx}|h) - m_x^\alpha h \right) + \chi \mu (1 - \chi) \alpha (\mu (1 - b_{hy}) m_y^{\alpha-1} h + (1 - \mu) (1 - b_{ly}) m_y^{\alpha-1} l) \quad (13)$$

where first term is the private benefit of the marginal learner becoming bilingual and the second term is the external benefit enjoyed by monoglots in community $y$ whose productivity is increased due to the marginal learner becoming bilingual. This implies that:

**Proposition 3** *In the pure human capital model, the net social benefit of the marginal second language learner at the equilibrium is positive.*

In the absence of the productivity benefit to the other language community, the first welfare theorem would apply and the net social benefit at the margin would equal

\(^{11}\)Since employers make zero profits in equilibrium, we only need to consider utilities in the social welfare calculation.

\(^{12}\) $\frac{\partial s_{hx}}{\partial b_{hx}} = h - G_x^{-1}(b_{hx}|h) g \left( G^{-1}(b_{hx}|\gamma) | \gamma \right) \frac{dg^{-1}(b_{hx}|h)}{db_{hx}} - h^\alpha m_x^{1-\alpha}$. And by the inverse function theorem: $g \left( G^{-1}(b_{hx}|\gamma) | \gamma \right) \frac{dg^{-1}(b_{hx}|h)}{db_{hx}} = 1$.  

15
to zero. In what follows, we will refer to this external benefit term as the network effect attributable to the marginal second language learner. We can characterize the network effect more concisely by treating $\theta$ and $b_j$ as random variables and defining:

$$E[\theta] \equiv \mu h + (1 - \mu) l, \quad E[b_j \theta] \equiv \mu b_{hj} h + (1 - \mu) b_j l$$

Then evaluating the sum $\partial SW/\partial b_{hx} + \partial SW/\partial b_{lx}$ at the equilibrium gives community $x$’s marginal network effect:

$$NE_x = \alpha \chi (1 - \chi) (E[\theta] - E[b_y \theta]) m_y^{\alpha-1}$$

Equivalently for community $y$:

$$NE_y = \alpha \chi (1 - \chi) (E[\theta] - E[b_x \theta]) m_x^{\alpha-1}$$

Equations (14) and (15) show how the network effects respond to changes in the model parameters. As $\chi \to 0$ or $\chi \to 1$, the effects vanish. This indicates that, when the linguistic population shares are highly uneven, the equilibrium approaches a social optimum.\textsuperscript{13} In this case, there are relatively few monoglot minority speakers who would benefit from a higher level of second language acquisition on the part of the majority, and the marginal benefit to majority monoglots to a higher level of second language acquisition by the minority is miniscule. An increase in the elasticity of language skill complementarity in production $\alpha$ increases the effect. Furthermore, the network effects are decreasing in the general level of bilingualism in the economy:

$$\chi B_x + (1 - \chi) B_y.$$ 

As a consequence, it is not possible to determine the sign of the derivatives $\partial NE_x/\partial \chi$ and $\partial NE_y/\partial \chi$ for all values of $\chi \in [0, 1]$.

\textsuperscript{13}This may be a local optimum since it has not been established that there is necessarily a unique global optimum.
2.2 Pure Signalling Model

In their analysis of 2001 Canada Census data, Christofides and Swidinsky (2010) found evidence that bilingual wage premia are paid to employees even when they make no use of the second language in the workplace. They suggest that this implies that employers use linguistic ability as a screen for otherwise unobservable ability.\(^{14}\)

Here we incorporate this insight by assuming that employers cannot observe the productivity of individual workers but can observe who is bilingual and the proportions of each ability type who become bilingual. So while language skills are unproductive, there is still a benefit to learning a second language in that it potentially serves as a signal of high ability since, on average, high types have lower costs of learning than low types. The assumption of weak correlation replaces the single-crossing property assumed in the canonical signalling literature (as in Spence (1973)). As a consequence, there is no issue of market unravelling in this game.

To draw a clear distinction between this version of the model and the one presented in the previous section, here we assume that \(\alpha = 0\), so that the production function of a type \(\theta\) individual is now:\(^{15}\)

\[
f(\theta) = \theta
\]  

(16)

In this case, language skills have no productive use.

The equilibrium is defined in terms of a set of conditional beliefs about worker productivities on the part of employers that, when translated into offered wages, are consistent with the observed reactions of workers. We let \(E[\theta|\delta = 0]\) and \(E[\theta|\delta = 1]\) represent the employers' beliefs about the productivity of monoglot and bilingual employees respectively. We examine three possible equilibria: pooling, where no individual of either type becomes bilingual; hybrid, where some proportion of each

\(^{14}\)This view is supported by Chorney (1998) who, in his analysis of bilingualism in employee recruitment, quoted a human resource manager at an Ontario high-tech company: "Ability to master a second language shows skills in other areas: social skills, perseverance. It's a good sign." p. 204.

\(^{15}\)In this section, we drop the subscript denoting the language community since there are no cross-community effects from language learning.
types becomes bilingual; and full separating, where only high types become bilingual.\footnote{The fourth possibility of a pooling equilibrium, where all workers become bilingual, is ruled out due to the infinite upper bound on the support of the learning cost distributions.}

It should be clear that full separation cannot be sustained as an equilibrium. If so, consistency requires that $E[\theta|\delta = 1] = h$ and $E[\theta|\delta = 0] = l$ and the wage premium would be $\Delta w = h - l > 0$. Since the support of $G(c|l)$ is $[0, \infty)$, this premium will always motivate some positive measure of low types to become bilingual.

A pooling equilibrium, where no individual of either type becomes bilingual, requires the wage premium $\Delta w = 0$. Thus by implication $E[\theta|\delta = 1] = E[\theta|\delta = 0]$. Since these beliefs must be confirmed in equilibrium, $E[\theta|\delta = 0] = E[\theta|\delta = 1] = E[\theta]$ where, as above, $E[\theta] \equiv \mu h + (1 - \mu) l$ is the unconditional expectation of the $\theta$ for the entire population.

Such beliefs will sustain a pooling equilibrium, however there is possibly an issue of stability. Note that if some worker is chosen at random from the population and induced to become bilingual, his expected productivity will indeed be $E[\theta]$. We then conclude that the "no-learning" pooling equilibrium is stable.

In the hybrid equilibrium, firm’s beliefs satisfy $E[\theta|\delta = 1] > E[\theta|\delta = 0]$. Then when $b_\theta > 0$ for both types these conditional expectations are:

$$E[\theta|\delta = 1] = \frac{E[b\theta]}{B} \quad (17)$$

and for the second term on the right hand side:

$$E[\theta|\delta = 0] = \frac{E[\theta] - E[b\theta]}{1 - B} \quad (18)$$

where $B = \mu b_h + (1 - \mu) b_l$ is the total rate of second language acquisition.

Then subtracting (18) from (17) and cross-multiplying the denominators gives the following formula for the wage premium:

$$\Delta w = \frac{Cov(b^*, \theta)}{B^* (1 - B^*)} \quad (19)$$
where \( \text{Cov}(b, \theta) \equiv E[b \theta] - E[b] E[\theta] \) and \( E[b] \equiv B \). The asterisks indicate equilibrium values.

The covariance term in the numerator reflects the value of the signalling game in distinguishing between high and low types. If the covariance between \( b^* \) and \( \theta \) was zero or negative, there would be no benefit to rewarding second language learning. To see this more clearly, the covariance can be expanded:

\[
\text{Cov}(b^*, \theta) = \mu (1 - \mu) (h - l) (b_h^* - b_l^*)
\]

(20)

so that, for any given \( B^* \), the wage premium is increasing in the differences \( h - l \) and \( b_h^* - b_l^* \). As \( b_h^* - b_l^* \) increases, the signalling game is more effective at sorting between high and low types. Although \( b_h^* \) and \( b_l^* \) are equilibrium values, since there is only one bilingual wage premium, their difference in equilibrium is determined by the stochastic dominance relation between the distributions \( G(c|h) \) and \( G(c|l) \). So the greater is the correlation between the second language learning abilities and non-linguistic productivities, the greater is the wage premium in this case. Furthermore as \( h - l \) increases, the higher is the benefit to employers of the sorting.

For an equilibrium where \( \Delta w > 0 \) is to exist, we require that the system of equations:

\[
b_\theta = G(\Delta w (b_h, b_l) | \theta), \quad \theta = l, h
\]

(21)

where \( \Delta w (b_h, b_l) = E[\theta|\delta = 1] - E[\theta|\delta = 0] \) from equations (17) and (18), has a solution \( b_h^* > 0 \) and \( b_l^* > 0 \). Since (21) is a system of fixed point equations mapping \([0, 1]^2\) into itself, there is always a solution where \( b_h^* \in [0, 1] \) and \( b_l^* \in [0, 1] \). However, there may or may not be a fixed point solution where both values \( b_h^* \) and \( b_l^* \) are positive.

Whenever more than one equilibrium exists, they can be ranked in terms of their associated social welfare levels. In particular, social welfare is strictly decreasing in the overall rates of second language acquisition.

**Proposition 4** In the pure signalling model, whenever more than one equilibrium exists, the highest level of social welfare is attained in the pooling equilibrium.
In other words, social welfare is maximized when no one becomes bilingual. The intuition is straightforward: bilingualism has no productive purpose other than sorting workers into wage categories. When some workers become bilingual, the sorting process will not change the sum of wages and resources will be expended.

2.3 Mixed Human Capital - Signalling Model

In this section, we incorporate the polar cases analyzed in the two previous sections into a general model where language skills both are productive and serve as a signal of unobservable ability. So as in the pure signalling model, $\theta$ is treated as a random variable from the point of view of employers. Furthermore, there exists the possibility of a "no-learning" pooling equilibrium. Firms may hold the belief that there is no difference in productivity between bilingual and monoglot individuals. However, provided that firms have some knowledge of the production technology, this belief is not rational, nor is this equilibrium welfare maximizing in the sense described above. On this basis we rule out the possibility of a no learning equilibrium in what follows.

Following the arguments made in the previous section, the equilibrium distribution of language skills is given by the solution $\{b^*_{hx}, b^*_{lx}, b^*_h, b^*_l\}$ to the system of equations:

$$b_{0j} = G_j (\Delta w_j | \theta), \quad \theta = l, h, \quad j = x, y$$ (22)

where the wage premia are determined by Bayes’ rule:

$$\Delta w_j = \frac{\mu b_{hj} l + (1 - \mu) b_{lj} l}{\mu b_{hj} + (1 - \mu) b_{lj}} - \frac{m^*_j \mu h + (1 - \mu) l - (\mu b_{hj} h + (1 - \mu) b_{lj} l)}{1 - (\mu b_{hj} + (1 - \mu) b_{lj})}, \quad j = x, y$$ (23)

We note that, in contrast to the pure human capital model in section 2.1, there is a single wage premium (i.e., not type-specific) for each community, and in contrast to the pure signalling model in section 2.2, the equilibrium in each community depends on the rates of second language learning in the other community.

Despite the informational asymmetry in this version of the model, the comparative static results for the full information case also apply here. The bilingual wage premium and the rate of bilingual knowledge will always be higher in the minority
language community. Furthermore, the stochastic dominance relation in the cost distributions imply that a higher proportion of high ability types will become bilingual than low ability types in both communities.

The net social benefit of high ability type marginal learners in community $x$ is:

$$\frac{\partial SW}{\partial b_{hx}} = \chi \mu \left( \Delta w_x - G^{-1}(b_{hx}|h) \right)$$

$$+ \chi \mu b_{hx} + (1 - \mu) b_{lx} \frac{\partial w_x^b}{\partial b_{hx}}$$

$$+ \chi (\mu (1 - b_{hx}) + (1 - \mu) (1 - b_{lx})) \frac{\partial w_x^m}{\partial b_{hx}}$$

$$+ \chi \mu (1 - \chi) \alpha \mu (1 - b_{hy}) + (1 - \mu) (1 - b_{ly}) \frac{\partial w_y^m}{\partial b_{hx}}$$

As with expression (13), the first term is the private benefit of becoming bilingual to the marginal learner which is equal to zero in equilibrium by substituting in (22). The term on the fourth line is the network effect which increases the productivity of monoglots in community $y$ and is strictly positive.

The two middle terms represent a signalling effect imposed by the marginal high type learner on members of his own community. Since workers are receiving wages corresponding to their expected productivity conditional on language skills, the marginal high type learner drives up the equilibrium bilingual wage and drives down the equilibrium monoglot wage. Conversely, the marginal low type learner drives up the monoglot wage and drives down the bilingual wage. In order to untangle the net effect, it is convenient to calculate the welfare effects of both the marginal low and high types in conjunction. In fact, for the purposes of policy analysis, it is the average marginal contribution that matters since presumably the government would be unable to observe abilities any better than firms. This leads to the conclusion:

**Proposition 5** In the mixed human capital - signalling model, marginal second language learners, on average, decrease the sum of utilities of their own language community and increase the sum of utilities of the other language community. Thus the sign of the average net social benefit is ambiguous.
The proof shows that on balance, the sum of signalling effects in community \( x \) is given by:

\[
SE_x = -\chi \frac{\text{Cov} (b^*_x, \theta)}{B^*_x (1 - B^*_x^\alpha)} \left( 1 - B^*_x \left( 1 - \left( \chi + (1 - \chi) B^*_y^\alpha \right) \right) \right)
\]  

(25)

The corresponding signalling effect for community \( y \) is:

\[
SE_y = -(1 - \chi) \frac{\text{Cov} (b^*_y, \theta)}{B^*_y (1 - B^*_y^\alpha)} \left( 1 - B^*_y \left( 1 - \left( (1 - \chi) + \chi B^*_x^\alpha \right) \right) \right)
\]  

(26)

As long as there is a positive bilingual wage premium in both communities, both (25) and (26) will be strictly negative. This is because the wage premium will induce some positive measure of both types to become bilingual. Due to the stochastic dominance relation between the cost distributions, the covariance term will be positive. The adverse social impact of the marginal learner is due to a mismatch between private and social values; in equilibrium, the marginal cost is set equal to the average benefit (the wage premium) while social welfare would be optimized if the marginal cost where set equal to the marginal benefit.

The reason that the summed signalling effects are negative has to do with how marginal learners influence the wage distribution. The marginal learners’ average productivity is less than the average productivity of all bilingual individuals and greater than the average productivity of all monoglots. So their learning decision drives down the expected productivities, and hence wages, of both groups.

In the special case where language skills are unproductive (\( \alpha = 0 \)) as presented in section (2.2), the signalling effect for each community reduces to:

\[
-\frac{\text{Cov} (b^*_j, \theta)}{B^*_j (1 - B^*_j^\alpha)}, \quad j = x, y
\]  

(27)

times the community’s population share. Since the wage premium is equal to this covariance term in equilibrium, the signalling effect is minimized when the premium is equal to zero and no one becomes bilingual.
When language skills are productive, the signalling effects also depend on the rate of second language acquisition in the other community. In particular $\partial SE_x/\partial B^*_y > 0$ and $\partial SE_y/\partial B^*_x > 0$, which implies that bilingualism has the second-order benefit on the other community of mitigating the welfare loss associated with signalling.

The formulas for the network effects in this case are as they were in the full information case (equations 14 and 15). In general, it cannot be claimed that either the signalling or network effects on social welfare will dominate in any given equilibrium. However, the results that the network effects vanish as $\chi \to 0$ and as $\alpha \to 0$ remain in force. The same cannot be said of the signalling effects: the sum of equations (25) and (26) may be far from zero even in the case of highly uneven population shares. Rather it is the covariances between the rates of second language learning $b^*_j$ and the ability types $\theta$ that dictate the how adverse the effects of signalling on social welfare will be. In particular, if either the correlation between language learning abilities and productivity is weak or the productivity difference between high and low types is small then the signalling effect will be small. If this is the case and there is a relatively large language minority, then we would expect the network effects to dominate and that social welfare could be improved by increasing the general level of bilingualism in the economy. The converse situation is true if the correlation between learning abilities and productivity is weak and the productivity difference between low and high types is large or the minority is relatively small. Then we would expect the signalling effects to dominate and that social welfare could be improved by reducing the level of bilingualism in the economy.

### 2.4 Summary

Table 2 presents a summary of results from the three models presented above. Inspection of the formulas for the bilingual wage premia show how the pure human capital model and the pure signalling model are limiting special cases of the mixed human capital - signalling model presented in section 2.3.

First to show that the pure human capital model is a special case, consider the
wage premium for community $x$ in the human capital - signalling model:

$$
\Delta w_x = \frac{E [b_x \theta]}{B_x} - m_x^\alpha \frac{E [\theta] - E [b_x \theta]}{(1 - B_x)}
$$

(28)

then set $\mu = 1$. This effectively eliminates the informational asymmetry between workers and firms since all workers are high types. Then because $E [b_x \theta] = b_{hx}h$, $E [\theta] = h$ and $B_x = b_{hx}$ when $\mu = 1$, the wage premium is:

$$
\Delta w_x = \frac{b_{hx}h}{b_{hx}} - m_x^\alpha \frac{h - b_{hx}h}{(1 - b_{hx})}
= (1 - m_x^\alpha)h
$$

(29)

as in the pure human capital model. By the same argument, setting $\mu = 0$ yields the wage premium for low types.

Then, to show that the pure signalling model is a special case, when $\alpha = 0$, the wage premium in the human capital - signalling model is:

$$
\Delta w_x = \frac{E [b_x \theta]}{B_x} - \frac{E [\theta] - E [b_x \theta]}{(1 - B_x)}
$$

(30)

which after cross-multiplying becomes:

$$
\Delta w_x = \frac{Cov (b_x, \theta)}{B_x (1 - B_x)}
$$

(31)

which is the wage premium in the pure signalling model.
Table 2: Summary and Comparison of Results from the Theoretical Models

<table>
<thead>
<tr>
<th>Model:</th>
<th>Pure Human Capital</th>
<th>Pure Signalling</th>
<th>Human Capital - Signalling</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wage Premium</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta w_{hx}$</td>
<td>$(1 - m_x^\alpha)h$</td>
<td>0 or $\frac{Cov(b_x, \theta)}{B_x(1 - B_x)}$</td>
<td>$E[b_x, \theta] - m_x^\alpha E[\theta] - E[b_x, \theta]$</td>
</tr>
<tr>
<td>$\Delta w_{hy}$</td>
<td>$(1 - m_y^\alpha)h$</td>
<td>0 or $\frac{Cov(b_y, \theta)}{B_y(1 - B_y)}$</td>
<td>$E[b_y, \theta] - m_y^\alpha E[\theta] - E[b_y, \theta]$</td>
</tr>
<tr>
<td>$\Delta w_{lx}$</td>
<td>$(1 - m_x^\alpha)l$</td>
<td>0 or $\frac{Cov(b_y, \theta)}{B_y(1 - B_y)}$</td>
<td>$E[b_y, \theta] - m_y^\alpha E[\theta] - E[b_y, \theta]$</td>
</tr>
<tr>
<td>$\Delta w_{ly}$</td>
<td>$(1 - m_y^\alpha)l$</td>
<td>0 or $\frac{Cov(b_y, \theta)}{B_y(1 - B_y)}$</td>
<td>$E[b_y, \theta] - m_y^\alpha E[\theta] - E[b_y, \theta]$</td>
</tr>
<tr>
<td>Comparative Statics</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\partial B_x / \partial \chi$</td>
<td>-</td>
<td>n.a.</td>
<td>-</td>
</tr>
<tr>
<td>$\partial B_y / \partial \chi$</td>
<td>+</td>
<td>n.a.</td>
<td>+</td>
</tr>
<tr>
<td>$\partial \Delta w_{xz} / \partial \chi$</td>
<td>-</td>
<td>n.a.</td>
<td>-</td>
</tr>
<tr>
<td>$\partial \Delta w_{yz} / \partial \chi$</td>
<td>+</td>
<td>n.a.</td>
<td>+</td>
</tr>
<tr>
<td>$\partial (\Delta w_{xz} - \Delta w_{yz}) / \partial \chi$</td>
<td>-</td>
<td>n.a.</td>
<td>-</td>
</tr>
<tr>
<td>$\partial (\Delta w_{xz} - \Delta w_{yz}) / \partial \Gamma$</td>
<td>+</td>
<td>n.a.</td>
<td>+</td>
</tr>
<tr>
<td>Welfare Effects</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\partial SW / \partial B_j$, $j = x, y$</td>
<td>+</td>
<td>n.a.</td>
<td>?</td>
</tr>
<tr>
<td>$NE_x$</td>
<td>$\alpha \chi (1 - \chi) (E[\theta] - E[b_x, \theta]) m_x^{\alpha - 1}$</td>
<td>n.a.</td>
<td>$\alpha \chi (1 - \chi) (E[\theta] - E[b_x, \theta]) m_x^{\alpha - 1}$</td>
</tr>
<tr>
<td>$NE_y$</td>
<td>$\alpha \chi (1 - \chi) (E[\theta] - E[b_y, \theta]) m_y^{\alpha - 1}$</td>
<td>n.a.</td>
<td>$\alpha \chi (1 - \chi) (E[\theta] - E[b_y, \theta]) m_y^{\alpha - 1}$</td>
</tr>
<tr>
<td>$SE_x$</td>
<td>n.a.</td>
<td>$\frac{Cov(b_x, \theta)}{B_x(1 - B_x)}$</td>
<td>$\frac{Cov(b_x, \theta)}{B_x(1 - B_x)} (1 - B_x (1 - m_x^\alpha))$</td>
</tr>
<tr>
<td>$SE_y$</td>
<td>n.a.</td>
<td>$\frac{Cov(b_y, \theta)}{B_y(1 - B_y)} (1 - B_y (1 - m_y^\alpha))$</td>
<td>$\frac{Cov(b_y, \theta)}{B_y(1 - B_y)} (1 - B_y (1 - m_y^\alpha))$</td>
</tr>
</tbody>
</table>
3 Conclusion

The preceding analysis has developed a model of second language acquisition where the incentive to become bilingual is driven by the market demand for language skills. Thus we implicitly assume that individuals are primarily driven to learn a second language by pecuniary motives. This is obviously a simplification made in order to focus on the matter at hand. It should be recognized that other benefits of bilingualism include expanded social and cultural opportunities.

Bilingual ability is rewarded in the model both due to its productive value and due to its capacity to signal unobservable traits. The equilibrium level of second language acquisition may either exceed or fall short of the welfare maximizing level of second language acquisition depending on whether network or signalling welfare effects dominate. Network externalities increase as: a) the size of the minority language community is relatively large and b) the elasticity of language skill complementarity in production increases.

Thus as the linguistic diversity of a population is higher or the structure of the economy puts more emphasis on language complementarities, language polices that increase the rate of second language acquisition may be welfare-improving. Under the opposite circumstances, where second language acquisition serves mainly as a signal, social welfare could be increased by discouraging second language acquisition. Real world language policies affect the costs and benefits of language use and acquisition in a multitude of ways. Examples of such policies include educational policies that lower marginal costs of learning, and labour market policies that increase benefits. Official recognition of minority languages frequently results in government services being made available in the language in question. In this case, there is both a reduced incentive to become bilingual and a potential for greater employment opportunities for minority language speakers. While language policies in practice are almost never motivated by concerns about productive efficiency, this analysis has contributed a number of insights about the economic dimensions of language policy.
Appendix

Proof of Proposition 1. The proof relies on the intermediate value theorem. Define the inverse of $B_x (B_y)$:

$$H (B_x) \equiv B_x^{-1} (B_y).$$

(32)

and:

$$\psi (B_x) \equiv H (B_x) - B_y (B_x).$$

(33)

Note that $\psi (B_x)$ is defined over the domain $[0, \overline{B}_x]$ where $\overline{B}_x$ satisfies:

$$\overline{B}_x = B_x (0) < 1$$

(34)

Then to establish proof of uniqueness we need to show that:

$$\psi (0) > 0$$

$$\psi (\overline{B}_x) < 0$$

and that under assumption A.4., $\psi (B_x)$ is strictly decreasing. This will imply that there is a single value of $B_x$ that satisfies $\psi (B_x) = 0$.

First observe that:

$$\mu G_x ((1 - [m_x (\chi, B_y)]^\alpha) h|h) + (1 - \mu) G_x ((1 - [m_x (\chi, B_y)]^\alpha l)|l) = 0$$

(35)

only when $m_x (\chi, B_y) = 1$ or, equivalently, $B_y = 1$, which implies that:

$$H (0) = 1.$$  

(36)

Furthermore:

$$B_y (0) < 1.$$  

(37)

Otherwise, if $B_y (0) = 1$, then both bilingual wage premia for community $y$ must be infinite in order for the highest learning cost individuals to become bilingual.
Inspection of equations 7 gives the upper bounds of the premia as $l$ and $h$. And since $l < h$, this implies that $\Delta w_{yl} < \infty$. So then:

$$\psi(0) \equiv 1 - B_y(0) > 0.$$  \hspace{1cm} (38)

By the same argument as above $\overline{B}_x < 1$, so:

$$B_y(\overline{B}_x) > 0.$$ \hspace{1cm} (39)

It remains to show that $\psi(B_x)$ is strictly decreasing under assumption A.4. This is so when:

$$\frac{dH(B_x)}{dB_x} - \frac{dB_y(B_x)}{dB_x} < 0$$

Differentiating equations 8 and 9 and rearranging gives:

$$\frac{dH(B_x)}{dB_x} = -\frac{1}{(\mu g_x(\cdot|h) + (1 - \mu) g_x(\cdot|l)) \alpha m_x^{\alpha-1} (1 - \chi)}$$ \hspace{1cm} (40)

and:

$$\frac{dB_y}{dB_x} = -(\mu g_y(\cdot|h) + (1 - \mu) g_y(\cdot|l)) \alpha m_y^{\alpha-1} \chi$$ \hspace{1cm} (41)

then the condition above becomes:

$$(\mu g_y(\cdot|h) + (1 - \mu) g_y(\cdot|l)) (\mu g_x(\cdot|h) + (1 - \mu) g_x(\cdot|l)) \alpha^2 m_x^{\alpha-1} m_y^{\alpha-1} \chi (1 - \chi) < 1$$

then using the definitions $E[g_x(c|\theta)] \equiv \mu g_x(c|h) + (1 - \mu) g_x(c|l)$ and $E[g_y(c|\theta)] \equiv \mu g_y(c|h) + (1 - \mu) g_y(c|l)$, the condition becomes:

$$E[g_x(c|\theta)] E[g_y(c|\theta)] \times \alpha^2 \times m_y^\alpha m_x^\alpha \times \frac{1 - \chi}{m_x m_y} < 1$$

and since $\alpha^2 \leq 1$, $m_y^\alpha m_x^\alpha \leq 1$, $\frac{1 - \chi}{m_x m_y} \leq 1$ and $\frac{1 - \chi}{m_x m_y} \leq 1$ it is sufficient that $E[g_x(c|\theta)] E[g_y(c|\theta)] < 1$ for the inequality to hold. 

**Proof of Proposition 2.** First observe that the following results hold from
equations 8 and 9:

\[
\frac{\partial B_x (B_y)}{\partial B_y} < 0, \quad \frac{\partial B_x (B_y)}{\partial \chi} < 0, \quad \frac{\partial B_x (B_y)}{\partial \mu} > 0 \quad (42)
\]

\[
\frac{\partial B_y (B_x)}{\partial B_x} < 0, \quad \frac{\partial B_y (B_x)}{\partial \chi} > 0, \quad \frac{\partial B_y (B_x)}{\partial \mu} > 0. \quad (43)
\]

The first statement in the proposition follows directly from the results above and the chain rule:

\[
\frac{d B_x^*}{d \chi} = \frac{\partial B_x}{\partial \chi} + \frac{\partial B_x}{\partial B_y} \frac{\partial B_y}{\partial \chi} < 0 \quad (44)
\]

and:

\[
\frac{d B_y^*}{d \chi} = \frac{\partial B_y}{\partial \chi} + \frac{\partial B_y}{\partial B_x} \frac{\partial B_x}{\partial \chi} > 0. \quad (45)
\]

The second statement follows from the fact that the term \(1 - m_x^\alpha = 1 - (\chi + (1 - \chi) B_y)^\alpha\) goes to zero as \(\chi\) approaches one and the term \(1 - m_y^\alpha = 1 - ((1 - \chi) + \chi B_x)^\alpha\) goes to zero as \(\chi\) approaches zero.

**Proof of Proposition 3.** The equilibrium rate of bilingualism for high types in community \(x\) is: \(b^*_{hx} = G ((1 - m_x^\alpha) h|h)\). Thus the first line of 13 is:

\[
\chi \mu \left( h - G^{-1} (b^*_{hx}|h) - m_x^\alpha h \right) = \chi \mu (h - h (1 - m_x^\alpha) - m_x^\alpha h) \quad (46)
\]

\[= 0. \]

What remains, the external benefit term, is strictly positive. ■

**Proof of Proposition 4.** In the "no learning" pooling equilibrium, the per capita wage bill for firms for each language community is equal to the expected productivity of a given worker: \(\mu h + (1 - \mu) l\). Since no one becomes bilingual, no resources are expended on language learning and all workers receive the same wage, so the wage bill is also equal to the social welfare of the language group in question. In any hybrid equilibrium, the per capita wage bill is equal to:

\[
B \frac{E[\theta]}{B} + (1 - B) \frac{E[\theta] - E[\theta]}{1 - B} = E[\theta] \quad (47)
\]
which is just the same as in the pooling equilibrium. However, in this case, resources are being expended on language learning. Thus the sum of utilities must be higher under the pooling equilibrium.

**Proof of Proposition 5.** We consider net social benefit of the marginal learners of each type in community \( x \):

\[
\frac{\partial SW}{\partial b_{hx}} = \chi \left( \mu \frac{\partial s_{hx}}{\partial b_{hx}} + (1 - \mu) \frac{\partial s_{lx}}{\partial b_{hx}} \right) + \left( 1 - \chi \right) \left( \mu \frac{\partial s_{hy}}{\partial b_{hx}} + (1 - \mu) \frac{\partial s_{ly}}{\partial b_{hx}} \right) \quad (48)
\]

\[
\frac{\partial SW}{\partial b_{lx}} = \chi \left( \mu \frac{\partial s_{hx}}{\partial b_{lx}} + (1 - \mu) \frac{\partial s_{lx}}{\partial b_{lx}} \right) + \left( 1 - \chi \right) \left( \mu \frac{\partial s_{hy}}{\partial b_{lx}} + (1 - \mu) \frac{\partial s_{ly}}{\partial b_{lx}} \right)
\]

Let:

\[
w_{b}^{x} = E \left[ \theta^{[\delta = 1]} \right] = \frac{\mu b_{hx} h + (1 - \mu) b_{lx} l}{\mu b_{hx} + (1 - \mu) b_{lx}} \quad (49)
\]

and

\[
w_{m}^{x} = E \left[ m_{x}^{\theta} \theta^{[\delta = 0]} \right] = \frac{m_{x} \mu h + (1 - \mu) l - (\mu b_{hx} h + (1 - \mu) b_{lx} l)}{1 - (\mu b_{hx} + (1 - \mu) b_{lx})} \quad (50)
\]

So:

\[
\frac{\partial s_{hx}}{\partial b_{hx}} = w_{b}^{x} - G^{-1}(b_{hx} | h) - w_{m}^{x} + b_{hx} \frac{\partial w_{b}^{x}}{\partial b_{hx}} + \left[ 1 - b_{hx} \right] \frac{\partial w_{m}^{x}}{\partial b_{hx}} \quad (51)
\]

\[
\frac{\partial s_{lx}}{\partial b_{hx}} = b_{lx} \frac{\partial w_{b}^{x}}{\partial b_{hx}} + \left[ 1 - b_{lx} \right] \frac{\partial w_{m}^{x}}{\partial b_{hx}}
\]

\[
\frac{\partial s_{hy}}{\partial b_{hx}} = \left[ 1 - b_{hy} \right] \frac{\partial w_{m}^{x}}{\partial b_{hx}}
\]

\[
\frac{\partial s_{ly}}{\partial b_{hx}} = \left[ 1 - b_{ly} \right] \frac{\partial w_{m}^{y}}{\partial b_{hx}}
\]
and:

\[
\begin{align*}
\frac{\partial s_{l_x}}{\partial b_{l_x}} &= w_x^b - G^{-1}(b_{l_x}|l) - w_x^m + b_{l_x} \frac{\partial w_x^b}{\partial b_{l_x}} + [1 - b_{l_x}] \frac{\partial w_x^m}{\partial b_{l_x}} \quad (52) \\
\frac{\partial s_{h_x}}{\partial b_{l_x}} &= b_{h_x} \frac{\partial w_x^b}{\partial b_{l_x}} + [1 - b_{h_x}] \frac{\partial w_x^m}{\partial b_{l_x}} \\
\frac{\partial s_{h_y}}{\partial b_{l_x}} &= [1 - b_{h_y}] \frac{\partial w_y^m}{\partial b_{l_x}} \\
\frac{\partial s_{l_y}}{\partial b_{l_x}} &= [1 - b_{l_y}] \frac{\partial w_y^m}{\partial b_{l_x}}.
\end{align*}
\]

In equilibrium \(w_x^b - G^{-1}(b_{h_x}|h) - w_x^m = 0\) and \(w_x^b - G^{-1}(b_{l_x}|l) - w_x^m = 0\), so we classify what remains into signalling effects:

\[
\begin{align*}
\frac{\partial S_x}{\partial b_{h_x}} &= \chi \left( \mu \left( \frac{\partial s_{h_x}}{\partial b_{h_x}} + (1 - \mu) \frac{\partial s_{l_x}}{\partial b_{h_x}} \right) \\
&= \chi \left( \mu \left( b_{h_x} \frac{\partial w_x^b}{\partial b_{h_x}} + [1 - b_{h_x}] \frac{\partial w_x^m}{\partial b_{h_x}} \right) + (1 - \mu) \left( b_{l_x} \frac{\partial w_x^b}{\partial b_{l_x}} + [1 - b_{l_x}] \frac{\partial w_x^m}{\partial b_{l_x}} \right) \right) \\
\frac{\partial S_x}{\partial b_{l_x}} &= \chi \left( \mu \left( \frac{\partial s_{h_x}}{\partial b_{l_x}} + (1 - \mu) \frac{\partial s_{l_x}}{\partial b_{l_x}} \right) \\
&= \chi \left( \mu \left( b_{h_x} \frac{\partial w_x^b}{\partial b_{l_x}} + [1 - b_{h_x}] \frac{\partial w_x^m}{\partial b_{l_x}} \right) + (1 - \mu) \left( b_{l_x} \frac{\partial w_x^b}{\partial b_{l_x}} + [1 - b_{l_x}] \frac{\partial w_x^m}{\partial b_{l_x}} \right) \right),
\end{align*}
\]

and network effects:

\[
\begin{align*}
\frac{\partial S_y}{\partial b_{h_x}} &= (1 - \chi) \left( \mu \left( \frac{\partial s_{h_y}}{\partial b_{h_x}} + (1 - \mu) \frac{\partial s_{l_y}}{\partial b_{h_x}} \right) \\
&= (1 - \chi) \left( \mu \left[ 1 - b_{h_y}^* \right] \frac{\partial w_y^m}{\partial b_{h_x}} + (1 - \mu) \left[ 1 - b_{l_y}^* \right] \frac{\partial w_y^m}{\partial b_{h_x}} \right) \\
\frac{\partial S_y}{\partial b_{l_x}} &= (1 - \chi) \left( \mu \left( \frac{\partial s_{h_y}}{\partial b_{l_x}} + (1 - \mu) \frac{\partial s_{l_y}}{\partial b_{l_x}} \right) \\
&= (1 - \chi) \left( \mu \left[ 1 - b_{h_y}^* \right] \frac{\partial w_y^m}{\partial b_{l_x}} + (1 - \mu) \left[ 1 - b_{l_y}^* \right] \frac{\partial w_y^m}{\partial b_{l_x}} \right).
\end{align*}
\]
Adding together the signalling effects of each type:

\[
\frac{\partial S_x}{\partial b_{hx}} + \frac{\partial S_x}{\partial b_{lx}} = \chi B_x^* \left( \frac{\partial w^b_x}{\partial b_{hx}} + \frac{\partial w^b_x}{\partial b_{lx}} \right) + \chi (1 - B_x^*) \left( \frac{\partial w^m_x}{\partial b_{hx}} + \frac{\partial w^m_x}{\partial b_{lx}} \right)
\]

\[
= \chi B_x^* \left( \frac{\mu (1 - \mu) (h - l) b_{hx}^* - \mu (1 - \mu) (h - l) b_{lx}^*}{(B_x^*)^2} \right) + \chi (1 - B_x^*) \left( -m_x \mu (1 - \mu) (h - l) (1 - b_{hx}^*) \frac{1}{(1 - B_x^*)^2} + m_x \mu (1 - \mu) (h - l) (1 - b_{lx}^*) \frac{1}{(1 - B_x^*)^2} \right)
\]

\[
= -\chi \mu (1 - \mu) (h - l) (b_{hx}^* - b_{lx}^*) \left( \frac{1}{B_x^*} + \frac{m_x}{1 - B_x^*} \right)
\]

\[
= -\chi \frac{\text{Cov}(b_x^*, \theta)}{B_x^* (1 - B_x^*)} (1 - B_x^* (1 - (\chi + (1 - \chi) B_y^*)^\alpha)) < 0.
\]

### References


