News Shocks and the Term Structure of Interest Rates: 
A Challenge for DSGE Models*

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Abstract 

News shocks about future increases in Total Factor Productivity (TFP) lead to a large and persistent drop in inflation and the Federal Funds rate while driving up the slope of the term structure of interest rates (Kurmann and Otrok, 2011). In this paper, we first show that a monetary DSGE model with a standard parametrization along the lines of Smets and Wouters (2007) is unable to replicate these dynamics. We then formally estimate the model with a limited-information procedure that is designed to match as closely as possible the impulse responses of key macroeconomic aggregates, inflation and the term structure to TFP news shocks. When we restrict parameters to economically meaningful values, the model is unable to generate the large drop in inflation and the Federal Funds rate that causes the slope of the term structure to increase. This failure to quantitatively account for the impact of TFP news shocks represents a challenge for modern DSGE models because TFP news shocks explain a significant portion of the variation in inflation, the Federal Funds rate and the term structure of interest rate as well as medium-run fluctuations in future real activity.

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1 Introduction

The macroeconomic literature has witnessed a resurgence of the idea that economic fluctuations are driven by news about future changes in fundamentals. Originally proposed by Pigou (1927), the idea has been resuscitated in a widely-cited paper by Beaudry and Portier (2006) who argue that anticipations of future increases in Total Factor Productivity (TFP) are captured by stock prices and account for a substantial portion of the variation in real activity. More recently, Kurmann and Otrok (2011) document that movements in the slope of the U.S. term structure essentially reflect news about future TFP innovations. Specifically, a positive TFP news shock leads to a large drop in inflation, to which monetary policy reacts with a sharp and persistent decrease in the Federal Funds rate. As a result, the slope of the term structure steepens, thus providing a new explanation for why increases in the slope generally predict future economic growth.\footnote{Barsky and Sims (2010) also document that TFP news shocks lead to a sharp and persistent drop in inflation.}

While there remains controversy about the importance of TFP news shocks for fluctuations in real activity at business cycle frequencies (e.g. Barsky and Sims 2010), the strong and immediate response of inflation and asset prices to TFP news shocks seems to be a robust feature of the data. In this paper, we investigate to what extent a medium-scale DSGE model along the lines of Smets and Wouters (2007) can generate these responses; in particular the sharp fall of inflation and the Federal Funds rate together with the concomitant increase in the slope of the term structure. This exercise is important because medium-scale DSGE models have become a benchmark for policy analysis, both in academic work and in central banks. There is also great interest in incorporating asset prices in these models. As we show in Kurmann and Otrok (2011), TFP news shocks explain over 50% of the variations in the Federal Funds rate and the slope of the term structure, and determine a substantial portion of real economic activity at medium-run and growth frequencies. Hence, evaluating DSGE models along this particular dimension should be of prime interest.

Figure 1 provides a preliminary look at the ability of a medium-scale DSGE model to fit the macroeconomic dynamics conditional on a positive TFP news shock. The solid black lines display the empirical impulse responses computed with the VAR identification procedure in Kurmann and Otrok (2011), using post-war U.S. data. The dotted red lines show the theoretical impulse responses of the Smets and Wouters (2007) model with the parameters set to their full sample estimates. Leaving the details of the VAR identification procedure and the DSGE model for later, the figure shows that the model fails miserably to replicate the empirical VAR responses. In the data, the positive TFP news shock leads to a gradual
increase in TFP; generates a short temporary downturn followed by a gradual permanent increase in economic activity; and implies a sharp prolonged drop in inflation. Monetary policy accommodates with a more than proportional drop in the Federal funds rate; the slope of the term structure increases; and the long end of the term structure displays a sizable hump-shaped negative response. In the model, by contrast, the same TFP news shock has only a modest impact on real variables and barely moves inflation, the Federal funds rate and the term structure.

Figure 1: Impulse responses to a TFP news shock from empirical VAR (solid black lines and grey 68% confidence intervals) and the DSGE model calibrated to the Smets-Wouters (2007) estimates (dotted red lines).

This preliminary evaluation is admittedly unfair to the model since the estimation of Smets and Wouters (2007) is not conditioned on a TFP news shock and does not try to fit the term structure. Nonetheless, Figure 1 is instructive of just how far we will need to move to reconcile the model with the data. The paper proceeds in two steps to make progress. First, we use a limited-information estimation GMM procedure to search for the combination
of parameters that matches the VAR impulse responses to the TFP news shock as closely possible. We choose this procedure over full-information likelihood-based methods because we want to give the model the best possible shot at replicating the empirical evidence. Yet, we find that there is no reasonable set of parameters that generates impulse responses close to the ones implied by the VAR. A likelihood-based estimation with many shocks would yield an even worse fit. We view this failure as an important challenge for DSGE modeling, especially because the type of medium-scale model we use has become a benchmark for policy analysis, whether in academic work or at central banks. In a second step, we then explore several modifications to the benchmark model that are intended to help match the empirical responses to the TFP news shock.

The model by Smets and Wouters (2007) that serves as our benchmark includes sticky nominal price and wage setting, habit persistence in consumption, investment adjustment costs, variable capital utilization and fixed costs in production. Since the model is loglinearized and features homoscedastic innovations, term premia implied by the model are by definition constant (i.e. the expectations hypothesis holds). In this sense, it may not come as a big surprise that the model fails at generating large responses in the slope and the long end of the term structure. We therefore append a time-varying term premium part to the model. To do so we follow Ang and Piazzesi (2003) and many others in the finance literature and specify an affine no-arbitrage condition that generates time-varying risk as a linear function of state variables. Long bond yields can then be derived recursively as a function of expected future short yields and time variation in term premia. In order to maintain parsimony, we restrict risk to depend on two key variables of our DSGE model: expected inflation and expected changes in the marginal utility of consumption. Both of these variables have been shown in the finance literature to be important factors for the term structure. In contrast to that literature, however, the evolution of the two variables and thus risk is fully dictated by the solution to our linearized DSGE model. Furthermore, the parsimony of our setup restricts time variation in term premia in response to news shocks to depend on only two free parameters.\footnote{To our knowledge, the only paper that integrates an affine no-arbitrage bond pricing block into a DSGE framework is Hordahl, Tristani and Vestin (2006). However, their DSGE model is much more stylized than ours, the details of the specification are different from ours and they do not consider TFP news shocks.} We emphasize that this term premia is not the model-implied term premia. Instead it is an exploratory step in the direction of seeing what risk factors are being priced when there is a news shock. We hope this information adds a more positive conclusion to the paper in that we are able to suggest directions for future work. In our results, we report responses to news shocks under both pricing methods.\footnote{An alternative approach to generate time-varying term premia is to work with a non-linear version}
As noted above, we estimate the model parameters using a limited-information GMM technique as in Christiano, Eichenbaum and Evans (2005) with the objective of bringing the theoretical impulse responses as close as possible to the empirical counterparts implied by the VAR. For this approach to be coherent the DSGE model must be invertible in the sense of Fernandez-Villaverde, Rubio-Ramirez, Sargent, and Watson (2007). We show that for our specification of the TFP process the model is invertible and therefore has a VAR representation. Furthermore, we use simulation exercises to show that the VAR impulse responses to a TFP news shock provide a good approximation of impulse responses generated from the DSGE model. This suggests that the limited-information estimation we perform is a valid empirical testing method in this instance.

The key result coming out of our estimation exercise is that the model requires extreme nominal wage rigidity and an almost complete absence of price rigidity to generate a sizable drop in inflation on impact of the TFP news shock. Furthermore, there can be little or no investment adjustment cost and variable capital utilization should be relatively costless. Once we restrict the relevant parameters to values typically required for the model to replicate the dynamics with respect to standard business cycle shocks, the model fails to generate responses of inflation and real activity in line with the VAR evidence (i.e. responses similar to what we report in Figure 1 obtain). Furthermore, we find that monetary policy must react strongly to both inflation and output growth but not the output gap so as to replicate the large decrease in the Federal funds rate. But even if that is the case, the model is not capable of generating the persistent dynamics of the short-rate that imply the sizable and long-lasting increase in the expectational part of the term structure that we observe in the VAR. In our view, these findings hold important lessons on the determination of wages, the way nominal rigidities are specified as well as the slow diffusion of new technology throughout the economy. In order to build a structural macro-finance model consistent with the data these features of the standard model need to be rethought. At the same time, replicating the IRFs of asset prices to a news shock remains an important diagnostic in the process of building these models.

Our paper relates to several recent contributions in the VAR literature on TFP news of the DSGE model. This would have the advantage that the implied time-variations in term premia are model-consistent. Unfortunately, papers adopting such a non-linear approach have typically found it hard to generate sizable and sufficiently variable term premia. Additionally, estimating non-linear DSGE models of the size considered here is computationally very challenging. Donaldson, Johnson and Mehra (1990) and Den Haan (1995) are among the first to document the inability of basic DSGE models to generate large and volatile bond premia. More recently, Rudebusch and Swanson (2008, 2009) and Binsbergen et al. (2010) among others reexamine the same issue for DSGE models with long-memory habits or recursive preferences and found it typically hard to generate large and volatile bond premia.
shocks. First, the VAR results in Kurmann and Otrok (2010) broadly confirm the findings of Barsky and Sims (2010) that TFP news shocks do not lead to comovement in real variables and are therefore not a prime source of business cycle fluctuations. This stands in contrast to Beaudry and Portier (2006) who use another identification method for TFP news shock that results in both consumption and real activity to increase on impact of the TFP news shock. As we discuss, this would make it even more difficult for the model to fit the data since an increase in real activity on impact implies larger demand that puts upward-pressure on inflation.

Second, the paper is related to several studies that attempt to evaluate the importance of news shocks within a full-information likelihood-based context. For example, Schmitt-Grohe and Uribe (2010) conclude based on full-information estimates that TFP news shocks are a quantitatively important driver of business cycles. By contrast, Kahn and Tsoukalas (2010) show that Schmitt-Grohe and Uribe’s conclusion is highly sensitive to the exact nature of shocks present in the model. A virtue of our limited-information estimation approach is that it is not conditioned on the presence of other shocks, for which there is no clear consensus.

A third literature this paper addresses is the growing DSGE literature that investigates the linkages between various macroeconomic shocks and the term structure. For example, Wu (2006), Rudebusch and Wu (2008) and Bekaert, Cho, Moreno (2010) combine basic New-Keynesian models with no-arbitrage term structure models to investigate the role of various shocks on yields. Wu (2006) and Bekaert, Cho and Moreno (2010) conclude that monetary policy shocks explain a large portion of movements in the slope. This contrasts with De Graeve, Emiris and Wouters (2009) who use a larger DSGE model with many shocks (but no news shocks) and find that monetary policy shocks play a much smaller role for the slope. Instead, demand shocks, defined as innovations to the intertemporal consumption Euler equation, explain up to 50 percent of movements in the slope. We interpret our results with respect to these papers as follows. Wu (2006), Rudebusch and Wu (2008) and Bekaert, Cho and Moreno (2010) use relatively small models with few shocks. If these models are too stylized or the number of shocks is too small, the estimation may attribute movements in the short rate (which mostly drive the slope) to monetary shocks since this shock is simply the residual of an interest-rate rule. This is consistent with the results of De Graeve, Emiris and Wouters (2009) who argue, in addition, that term premia become quantitatively less important once expectations of future short rates are formed based on a larger DSGE model. We interpret their Euler equation shock that explains up to 50 percent of the slope as a measurement error left to be explained rather than a structural shock. Our news shock, in comparison, is one with a clear economic interpretation and
provides a 'deep' structural explanation for slope movements. Our results also suggest that while not dominant, variations in term premia remain an important source of term structure movements.

Section 2 of the paper briefly reviews the VAR identification procedure for identifying news shocks. Section 3 presents the DSGE model and Section 4 the results of the model. Section 5 concludes.

2 VAR identification of TFP news shocks

In this section, we briefly describe the VAR identification of TFP news shocks used by Barsky and Sims (2010b) and Kurmann and Otrok (2010). We then discuss the data used for the estimation and report results.

2.1 Identifying TFP news shocks

Consider the following process for TFP, as proposed by Beaudry and Portier (2006)

\[ a_t = v_t + D_t, \]

where \( a_t \) is the log of TFP; and \( v_t \) and \( D_t \) are two independent exogenous components. The component \( v_t \) captures potentially persistent but transitory surprise movements in TFP. The component \( D_t \) is non-stationary and assumed to follow a distributed lag process in past innovations; i.e. \( D_t = d(L)\eta_t \) with \( d(0) = 0 \). Innovations \( \eta_t \) are interpreted as news shocks about future productivity because they do not affect TFP contemporaneously, but only with a delay of one or more periods.

Rather than following the empirical approach of Beaudry and Portier (2006) who identify TFP news shocks with a mix of short- and long-run restrictions on stock prices and TFP, we adopt the more recent identification approach proposed by Barsky and Sims (2010b). In their procedure, TFP is placed in a VAR with a selection of other macroeconomic variables. The assumption underlying the identification procedure is that TFP is an exogenous process as in (1), driven by both contemporary and anticipated innovations (i.e. news). The identification of the news shock then consists of finding the orthogonal VAR innovation that explains most of the variations in TFP but has zero impact on TFP contemporaneously. This approach is a restricted version of Uhlig’s (2003) original idea of finding the exogenous shock(s) that, in a statistical sense, explain most of the fluctuations in output. It is implemented by extracting the shock that maximizes the amount of the forecast error variance (FEV) of TFP over
a given forecast horizon $k$ to $k$, with the additional side constraint that none of the FEV at $k = 1$ is explained. As Uhlig (2003) shows, this problem can be expressed as a simple Lagrangian with the TFP news shock being the first principal component (see details in the appendix).

As Barsky and Sims (2010b) discuss, this statistical identification approach has several desirable features. First, the approach allows but does not require that the TFP news shock has a permanent impact on TFP (i.e. $d(1) = 1$ in the above notation). Likewise, it does not make any restriction about the contemporary shock, nor that there are any common trends in the VAR variables. Second, the approach does not impose that there are only two shocks moving TFP, although in practice, this assumption turns out to be a good approximation. Third, because it is a partial identification method, the approach can be applied to large VAR systems without imposing additional and potentially invalid assumptions about other shocks.

### 2.2 Data

As in Kurmann and Otrok (2010), we specify a VAR that combines term structure and macroeconomic variables. For the term structure data we use two time series. The first is the Federal Funds rate. The second is the term spread which is measured as the difference between the 60-month Fama-Bliss unsmoothed zero-coupon yield from the CRSP government bonds files and the Federal Funds rate. We choose the 60-month yield as our long rate because it is available back to 1959:2, whereas longer-term yields such as the 120-month yield become available only in the early 1970s. We use the Federal Funds rate as the short term rate in order to be consistent with the macroeconomic model that we examine in Section 6. The DSGE model does not differentiate between the monetary policy rate and the short-end of the Treasury yield curve (e.g. a 3-month bill rate).\(^4\) To check for robustness, we ran our simulations with alternative measures of the slope and the short rate and found all of the main results to be unchanged.\(^5\)

For the macroeconomic data we use a measure of TFP, output, investment, consump-

\(^4\)This approximation seems reasonable since in practice, the Federal Funds rate and short-end bill rates move very closely together. More precisely, the correlation coefficient of the Federal Funds rate and the 3-month bill rate over the 1959:2-2005:2 period is 0.984. The Federal Funds rate is slightly more volatile and has a higher mean than the 3-month bill rate. For our VAR and DSGE exercises, these differences are not important.

\(^5\)There are two important alternative measures of the slope. First, we replaced the 60-month yield with the 120-month zero-coupon yield as computed by Gurkaynak, Sack and Wright (2007) and the Federal Funds rate by the 3-month bill rate. Second, we used a Nelson-Siegel style slope factor as computed in Diebold and Li (2006).
tion and inflation. The measure of TFP is a quarterly version of the series constructed by Basu, Fernald and Kimball (2006). This series exploits first-order conditions from a firm optimization problem to correct for unobserved factor utilization and is thus preferable to a simple Solow residual measure of TFP. The macro aggregates are all logged and in real chain-weighted terms. For inflation, we use the growth rate of the GDP deflator.

All of the macroeconomic series are obtained from the FRED II database of the St. Louis Fed and are available in quarterly frequency. The term structure and stock market data are available in daily and monthly frequency. We convert them to quarterly frequency by computing arithmetic averages over the appropriate time intervals. The sample period is 1959:2-2005:2 (with the start date limited by the availability of 60-month yield). Both the baseline VAR and the extended VAR are estimated in levels with 4 lags of each variable. To improve precision, we impose a Minnesota prior on the estimation and compute error bands by drawing from the posterior.

2.3 Results

The black solid lines in Figure 1 show the impulse responses to a TFP news shock. By definition, TFP does not react on impact of the shock. Thereafter, TFP increases gradually to its new permanent steady state. Output, consumption and investment also increase gradually to a new permanent level. On impact of the shock, consumption increases significantly whereas output and investment decline first. The real stock market index increases on impact and remains significantly higher for about four years before slowly returning back to its initial value. Finally, both inflation and the Federal Funds rate drop markedly on impact and remain persistently below their initial value for 15 to 20 quarters. Finally, the long rate declines only slightly.

Our VAR framework allows us to decompose the reaction of the long rate into variations due to term premia and expectations about future short rates (i.e. the Expectations Hypothesis). We can decompose the yield on a $T$-period yield $r_t^T$ (in our case the 60-month

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6Basu, Fernald and Kimball (2006) also make use of industry level data to correct for differences in returns to scale. Since this industry level data is available only on an annual basis, our quarterly TFP measure does not include this returns to scale correction. See Sims (2009) for details.

7We performed a battery of robustness checks with other macroeconomic variables including data that allowed us to estimate the VAR on monthly frequency. We discuss the responses of some of the added variables in the next section but note that none of the main conclusions is affected by the different changes in VAR specification. Also, we dropped the Minnesota prior and estimated the VAR with OLS instead, computing the error bands by bootstrapping from the estimated VAR. Details are available from the authors upon request.
yield) as
\[ r_t^T = \frac{1}{T} \sum_{i=0}^{T-1} E_t r_{t+i} + t p_t, \]  
(2)

where the \( E_t r_{t+i} \) are time \( t \) expectations of future short rates; and \( t p_t \) denotes term premia. The reaction of the long-rate with respect to TFP news shocks may be relatively small either because the Expectations Hypothesis part \( \frac{1}{T} \sum_{i=0}^{T-1} E_t r_{t+i} \) and the term premia part do not respond strongly or because variations in the two almost cancel each other out. This, in turn, determines the importance of term premia fluctuations for the reaction of our slope measure.

The technical difficulty with the decomposition in (6) is that term premia are inherently unobservable. Here, we follow Campbell and Shiller (1987, 1991) and use our larger VAR to compute expectations of short rates conditional on TFP news shocks. The term premia response to the TFP news shock is then simply the difference between the actual long rate response to the shock and the response as implied by the Expectations Hypothesis computed from the VAR. We will use impulse response of both the expectations hypothesis part and term premia in our evaluation of the DSGE model.

3 A DSGE-term structure model with TFP news shocks

We now evaluate how well a medium-scale DSGE model along the lines of Smets and Wouters (2007) can account for term structure movements in response to TFP news shocks. As in the existing literature, we linearize the model around the appropriately normalized steady states. This makes it easy to solve and estimate the model despite its relative complexity. Unfortunately, the linearization coupled with the assumption of homoscedastic innovations implies constant term premia. Since time varying term premia are important for understanding long bond yield movements we combine the linearized DSGE model with an affine formulation of the pricing kernel that allows for time-varying risk. Under no arbitrage, long bond yields can then be derived recursively as a combination of expected future short rates and time-variation in term premia. Movements in both components are governed entirely by the dynamics of the states variables from our DSGE model, which imposes considerable discipline on their dynamics.

3.1 Model

The macro part of the model is very similar to the one presented in Smets and Wouters (2007) and contains several real and nominal frictions. Specifically, the model features sticky nom-
inal price and wage setting that allows for indexation to lagged inflation, habit persistence in consumption, investment adjustment costs, variable capital utilization and fixed costs of production.

Compared to Smets and Wouters (2007), we augment the model with several additional elements intended to help the model match the VAR evidence. First and obviously, we need to introduce a TFP news shock. To this end, we specify TFP as an exogenous process with a stochastic trend, that is driven solely by a news shock; i.e.

\[ \mu_t = (1 - \gamma)\mu + \gamma \mu_{t-1} + \varepsilon_{\text{news}}^t, \]

where \( \mu_t = a_t - a_{t-1} \) is the growth rate of TFP and \( \varepsilon_{\text{news}}^t \) is the news shock. This process is a special case of the process in (1) and turns out to fit the evolution of TFP to the news shock extremely well. The news shock impacts actual TFP in period \( t \) but is known \( j \) periods in advance. The news shocks are i.i.d. processes with mean zero and variance \( \sigma_{\text{news}}^2 \). In our VAR, TFP begins to react after the first period after the news shock. So we set \( j = 1 \).

Second, we allow preferences to be of a more general form than in Smets and Wouters (2007)

\[ u(c_t, n_t) = \frac{(c_t - bc_{t-1} - \theta n_t^\eta s_t)^{1-\sigma} - 1}{1 - \sigma} \]

with

\[ s_t = (c_t - bc_{t-1})^\gamma s_{t-1}^{1-\gamma}. \]

This specification is as in Schmitt-Grohe and Uribe (2010) and accommodates both habit persistence in consumption (when \( b > 0 \)) and a limited role for short-run wealth effect as proposed by Jaimovich and Rebelo (2009). For \( \gamma = 1 \), (4) reduces to standard King-Plosser-Rebelo preferences in consumption and leisure with habit persistence. For \( \gamma \to 0 \), labor supply is not affected by wealth effects.

Third, we use a slightly different description of monetary policy than Smets and Wouters (2007) and specify the short-term nominal rate \( r_t \) as a function of expected inflation \( E_t \pi_{t+1} \), the output gap \( y_{\text{gap},t} \) and output growth \( \Delta y_t \) (rather than growth in the output gap)

\[ r_t = \rho r_{t-1} + (1 - \rho) [r + \theta E_t (\pi_{t+1} - \pi) + \theta_{y_{\text{gap}}} y_{\text{gap},t} + \theta_{\Delta y} (\Delta y_t - \Delta y)], \]

where the output gap is defined as the difference between actual output and potential output if there were no nominal price and wage rigidities.

For space reasons, we relegate a detailed description of the full model and its linearization to the appendix. The Rational Expectations equilibrium of the resulting system of equations
is computed using the numerical algorithm of King and Watson (1998) and can be expressed as a linear state-space system

$$\mathbf{Y}_t = \phi_Y + \Phi_Y \mathbf{S}_t$$

$$\mathbf{S}_t = \phi_S + \Phi_S \mathbf{S}_{t-1} + G \varepsilon_t,$$

where the $n \times 1$ vector $\mathbf{Y}_t$ contains the endogenous variables; the $k \times 1$ vector $\mathbf{S}_t$ contains the states; and the $k_x \times 1$ vector $\varepsilon_t$ contains the i.i.d. innovations to the exogenous shocks that we assume multivariate normal $(0, \mathbf{I})$.\(^8\)

For the term structure part of the model, notice that the short-term yield is part of the macro system and therefore included in the linear state-space system; i.e.

$$r_t = \delta_0 + \delta'_1 \mathbf{S}_t,$$

where $\delta_0$ and $\delta'_1$ contain the appropriate elements of $\phi_Y$ and $\Phi_Y$, respectively. The yield on a $T$—period discount bond is defined as

$$r_t^T = -\frac{\log P_t^T}{T},$$

where $P_t^T$ is the period-$t$ price of the bond with $P_t^0 = 1$. Under no arbitrage, this price satisfies

$$E_t \left[ M_{t+1}^0 P_{t+1}^{T-1} \right] = P_t^T,$$

where $M_{t+1}^0$ is the nominal pricing kernel. Following Ang and Piazzesi (2003) and many others in the latent factor no-arbitrage literature, we assume that the logarithm of this pricing kernel is described by

$$\log M_{t+1}^0 = -r_t - \frac{1}{2} \mathbf{A}_t' \mathbf{A}_t - \mathbf{A}_t' \varepsilon_{t+1},$$

where the $k_x \times 1$ vector $\mathbf{A}_t$ denotes the market price of risk associated with the different shocks in $\varepsilon_t$. Similar to Hordahl et al. (2007), these risk factor are assumed to follow an affine process in the states

$$\mathbf{A}_t = \lambda_0 + \lambda_1 \mathbf{S}_t,$$

with the $k_x \times 1$ vector $\lambda_0$ defining average risk; and the $k_x \times k$ matrix $\lambda_1$ defining how risk

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\(^8\)There are $k_y = k - k_x$ endogenous states (i.e. predetermined endogenous variables), which are ordered first in $\mathbf{S}_t$. Hence, $G$ is a $k \times k_x$ matrix with zeros in the upper $k_y \times k_x$ block and a diagonal matrix with the exogenous shocks’ standard deviations in the lower $k_x \times k_x$ block.
varies depending on the state of the economy. Given (6)-(12), bond prices can be computed recursively as linear functions of $S_t$ that can be decomposed into fluctuations due to expected future short rates (i.e. the Expectations Hypothesis) and time variations in term premia (see the appendix for details).

As shown by Wu (1996) and Bekaert, Cho and Moreno (2010), the pricing kernel implied by linearized DSGE models with homoscedastic innovations represent a special case of the formulation in (11) with $\lambda_0$ a function of the different structural parameters of the DSGE model and $\lambda_1 = 0$. In other words, the linearized DSGE model implies that risk and therefore term premia are constant. To allow for time-variation in term-premia we let $\lambda_1$ be non-zero. This potentially involves estimating the entire $k \times k$ matrix $\lambda_1$, which is large for our DSGE model. To make the estimation manageable, we impose two restrictions. The first restriction is that we let risk vary only with respect to two macro variables: expected inflation $E_t \pi_{t+1}$ and expected changes in marginal utility of consumption $E_t \Delta u_{c,t+1}$. This allows us to express risk associated with the news shock as

$$\lambda_t^{\text{news}} = \lambda_0^{\text{news}} + \lambda_{1,\pi} E_t \pi_{t+1} + \lambda_{1,\Delta u_c} E_t \Delta u_{c,t+1},$$

(13)

where $\lambda_{1,\pi}^{\text{news}}$ and $\lambda_{1,\Delta u_c}^{\text{news}}$ tell us how the price of risk with respect to the news shock reacts to changes in expected inflation and changes in the expected change of the marginal utility of consumption, respectively. Both of these variables are part of the state-space solution of our DSGE model in (6)-(7). The second restriction follows naturally from our limited information estimator in that the only exogenous shock we consider is the news shock. As the appendix shows in detail, the two restrictions together imply that $\lambda_{1,\pi}^{\text{news}}$ and $\lambda_{1,\Delta u_c}^{\text{news}}$ are the only additional free parameters to estimate. This imposes considerable discipline on our estimation (by comparison Ang and Piazzesi, 2003 estimate a total of 13 different coefficients in their formulation of $\Lambda_t$).

There are two important features of our modelling of risk. First, our formulation of time-varying risk can be motivated by the consumption-based asset pricing literature, which says that changes in risk over the business cycle must come from changes in the conditional covariances between inflation and the marginal utility of consumption (e.g. Piazzesi and Schneider, 2006). Establishing the link between risk and conditional covariances explicitly in the context of a non-linear DSGE model has been largely unsuccessful. Our formulation should therefore be considered as a basic test of whether variations in risk as a linear function of two macro variables are capable of generating quantitatively large term premia fluctuations. Second, the macro dynamics of our model as described by the state-space system in (6)-(7) are independent of time-variation in risk. Since risk depends on the macro states,
however, the joint estimation of both macro and term structure dynamics imposes discipline on the parameters of the macro model.

### 3.2 Estimation

We partition the parameters of the model into two groups. The first group consists of parameters that we calibrate to match long-run moments of the data. The second group is estimated to match the IRFs to a news shock as generated by the empirical VAR. All values reported are for a quarterly frequency.

Table 1 presents the calibrated parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Calibration</th>
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<tbody>
<tr>
<td>$\alpha$</td>
<td>Elasticity of production to labor</td>
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<td>$\beta$</td>
<td>Discount factor</td>
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<td>$\delta$</td>
<td>Depreciation rate</td>
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<td>$\theta_p$</td>
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<td>$\theta_w$</td>
<td>Elasticity of substitution across labor</td>
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<tr>
<td>$\eta$</td>
<td>Frisch elasticity of labor supply</td>
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</tr>
<tr>
<td>$\sigma$</td>
<td>Risk aversion</td>
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<tr>
<td>$\gamma$</td>
<td>Wealth effect parameter</td>
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</table>

The growth rate of output $\Delta y$ and TFP $\mu$ are set to match the average growth rate of real GDP and TFP in the data (1.86% and 1.29% annually for the 1959-2005 sample). The next four parameters imply a labor share of 0.675 in line with Gollin (2002), an average annualized quarterly real interest rate of 2.34% as measured in our data; an annual depreciation rate of 10 percent; and an average markup for final goods producers of 11% as reported by Basu and Fernald (1997). The elasticity of substitution across labor $\theta_w$ is set as in Smetts and Wouters (2007); the unit elasticity of labor supply $\eta$ is a compromise between values suggested in the microeconomic and macroeconomic literatures; and the fixed cost in production (not reported here) is set so that economy-wide net profits are zero as suggested by Basu and Fernald (1994) or Rotemberg and Woodford (1995).

The second group of parameters is estimated by minimizing a weighted distance between the model-implied IRFs to a news shock and the empirical counterparts from the VAR. Specifically, denote by $\hat{\Psi}$ a vector of empirical IRFs to a news shock over obtained from a VAR. Likewise, denote by $\Psi(\zeta)$ the same vector of IRFs implied by the model, where $\zeta$
contains all the structural parameters of the model. The estimator for the second group of parameters $\zeta_2 \subseteq \zeta$ is

$$\hat{\zeta}_2 = \arg \min_{\zeta_2} \left[ \hat{\Psi} - \Psi(\zeta) \right]^\prime \Omega^{-1} \left[ \hat{\Psi} - \Psi(\zeta) \right],$$

where $\Omega$ is a diagonal matrix with the sample variances of $\hat{\Psi}$ along the diagonal. This limited-information approach is the same than the one implemented by Christiano, Eichenbaum and Evans (2005) for a monetary policy shock. Here, we adapt it for our purposes by first estimating the parameters governing the response of TFP to an exogenous news shock and then, in a second step, by estimating the remaining structural model parameters such as to match the IRFs of other variables in the VAR. We adopt this two step approach because we want to evaluate the ability of our model to generate realistic term structure and macroeconomic dynamics to a news shock given the evolution of observed TFP.

On the macro side, we include the IRFs of TFP and all four macroeconomic variables of our extended VAR in the objective function. On the term structure side, we include the IRFs of the short rate, the long bond rate and the term spread as well as the IRFs of the long bond rate and the term spread implied by the Expectations Hypothesis (as computed in the previous section from the VAR). This provides us with a total of 10 empirical IRFs. For each of these IRFs, we include the entire 40 quarter horizon in the estimation criteria.

### 3.3 Accuracy of the VAR identification procedure

Before estimating the model, we need to show that our VAR impulse responses used in the estimation objective could at least in principle be a good approximation of the theoretical impulse responses from a DSGE model. This exercise involves two steps. In the first step, we need to show that the solution to our DSGE model is invertible in the sense of Fernandez-Villaverde, Rubio-Ramirez, Sargent, and Watson (2007); i.e. that TFP has an infinite VAR representation from which we can identify the news shock. To check this, we rewrite the state-space solution of the model in (7)-(6) as in Fernandez-Villaverde et al.

$$S_{t+1} = AS_t + B\varepsilon_{t+1}. \quad (14)$$

$$Y_t = CS_t + D\varepsilon_{t+1} \quad (15)$$

As Fernandez-Villaverde et al. show, a necessary condition for invertibility is that the eigenvalues of $(A - BD^{-1}C)$ are less than one in modulus. We check this for our model and find that the condition is satisfied. This result may come as a surprise because several papers
in the literature, among them Leeper, Walker and Yang (2008), argue that DSGE models with news shocks often have invertibility problems because anticipated shocks increase the number of unobserved states that is not spanned by the VAR variables. In our model, this same problem would arise if we specified a more basic news process instead of the slow diffusion process in (3) and if news shocks arose with more than one period in advance; i.e. \( j > 1 \). However, as we will see below, our TFP specification and the assumption that \( j = 1 \) provides an excellent match for the observed TFP dynamics after a news shock. Hence, we are justified using this particular process in (3).

Second, it is an open question whether the impulse responses of a truncated VAR of the type we use provides a good approximation of the true impulse responses. Following the recommendation of Chari, Kehoe, and McGrattan (2008), we therefore simulate a large number of data samples from our DSGE model for both the Smets-Wouters (2007) calibration and the unrestricted estimates reported in Table 1 and 2 below. We then compare these impulse responses to the theoretical impulse responses from the model. As in Barsky and Sims (2010) who perform a similar exercise on a much smaller DSGE model without nominal frictions, we find that the VAR identification does a good job in identifying TFP news shocks and tracing out the resulting impulse responses. Our VAR responses should therefore be considered as an appropriate target against which to test our DSGE model.

4 Results

We provide two estimates of the parameters of the model. The first is an unconstrained estimation where we let the estimated parameters in \( \zeta_2 \) take on any value within the theoretically admissible bounds. The second is a constrained estimation where we force a subset of parameters in \( \zeta_2 \) to not exceed values that are realistic economically or with respect to other estimates found in the literature. The first set of estimates is motivated by our desire to maximize the fit of the model. The second set is suggestive of the difficulties of matching the responses of a macro and finance variables to a news shock while maintaining an ability to match other known empirical facts.

Figure 4 plots the model IRFs implied by the unconstrained estimation and compares them to the IRFs from the VAR (with the grey-shaded areas demarking the 16%-84% error bands of the VAR responses). Overall, the estimated model does well in matching the responses of the different macro variables. As the plot for TFP shows, the stochastic growth process in (3) almost perfectly traces the gradual increase of TFP after the news shock. The model also matches closely the initial jump in consumption and the subsequent gradual
increase to the new balanced growth level. For output and investment, the model misses the initial drop in both variables and fails to generate the sharp increase in investment over the following periods.\textsuperscript{9} Over the longer run, however, the model matches the increase of both variables to the new balanced growth level. Likewise, the model generates the overall shape of the inflation response even though on impact of the news shock, the model implies a drop in inflation that is too modest.

For the term structure variables, the performance of our model is more mixed. The model is successful in generating the sizable jump in the spread and the gradual return back to its average value that we obtain from the VAR. In order to get this large initial jump, however, the model needs an increase in the term premium that is too large relative to its empirical counterpart. As a result, the long-bond yield initially responds with the wrong sign before matching the data. For the short rate, the model generates a sharp drop on impact of the shock but this drop is only half as large as in the VAR. Consequently, the initial response of the spread implied by the Expectations Hypothesis remains well below its VAR counterpart.

The estimated parameters that generate the IRFs in Figure 4 are reported in Table 2 in the column labeled 'unconstrained estimates'. The estimates $\kappa_p = 1$ and $\omega_p = 0$ indicate that the data favors a purely forward-looking New Keynesian Phillips curve (NKPC) with little price rigidity (i.e. with standard Dixit-Stiglitz goods differentiation, a coefficient of $\kappa_p = 1$ implies an average price duration of only 1.6 quarters).\textsuperscript{10} By contrast, the data requires an extreme degree of nominal wage rigidity with an estimated frequency of wage reoptimization of only $1 - \xi_w = 0.01$ per quarter and a degree of indexation for non-reoptimized wages to past inflation of $\omega_w = 0.70$. The main force behind these estimates is the sharp drop of inflation on impact of the news shock, which the model can generate only if inflation is a mainly forward-looking process that reacts strongly to current and future expected marginal cost (i.e. $\omega_p$ is small and $\kappa_p$ is large). Marginal cost, in turn, depends positively on wages

\textsuperscript{9}As we mentioned in Section 4, there is controversy about whether output and investment decreases on impact of a news shock. While important, this question is not the focus of our investigation. We therefore attach relatively little importance to whether our model is capable of generating a negative initial reaction of output and investment or not.

\textsuperscript{10}Since we use a limited information approach to estimate a relatively large number of parameters, it is not surprising that we face a number of weak identification issues. For the unconstrained estimation, this manifests itself in a marginal cost coefficient $\kappa_p$ that tends to wander off towards very large values without much improvement in the estimation objective and, consequently, very large associated standard errors. We therefore fix $\kappa_p = 1$ in this estimation. This does not change any of the conclusions. Also note that the inflation indexation parameter $\omega_p$ is estimated at its lower bound. Since it would not be meaningful to report a standard error at this boundary, we fix the parameter when computing standard errors for the other estimates. We adopt the same approach for any other parameter that is estimated at its respective lower or upper bound.
and negatively on TFP. After a news shock, the negative income effect on labor supply from consumption smoothing puts upward pressure on wages and thus on marginal cost. In subsequent periods, as the expected increase in TFP realizes, marginal cost falls. The drop in inflation on impact and the gradual response thereafter occurs only if there is a lot of wage rigidity (i.e. $\xi_w$ and $\omega_w$ large) so that the initial increase in marginal cost is relatively modest and its negative reaction after the TFP shock realizes is large.

The estimation also has strong implications for capital utilization and investment adjustment cost. The parameter governing the variability of capital utilization $\sigma_u$ is estimated close to its lower bound of 0, which implies that capital utilization is roughly proportional to the rental rate of capital.\(^\text{11}\) As Dotsey and King (2006) show, variable capital utilization reduces the sensitivity of marginal cost. Hence, the smaller the cost of utilization, the less pressure production exerts on marginal cost. This helps the model reconcile the large expansion of production with the persistent drop in inflation in the wake of the news shock. The investment adjustment cost parameter, in turn, is estimated at its lower bound of $S'' = 0$ (i.e. adjustment costs are zero in the vicinity of the steady state). This estimate is driven by the initial drop of investment and the need for little pressure on marginal cost on impact of the shock. If investment adjustment costs were large, then there would be a strong incentive to smooth investment, which in turn would put upward pressure on production and inflation.

Turning to monetary policy, the estimates $\rho = 0.0$ and $\theta_y = 2.92$ indicate that the Fed does not smooth its policy rate and reacts aggressively to inflation. Both parameter estimates are crucial to generate the sharp drop in the Federal Funds rate on impact.\(^\text{12}\) The estimates $\theta_{y\text{gap}} = -0.02$ and $\theta_{\Delta y} = 2.13$ imply that the Fed does not respond to the output gap but reacts strongly to output growth, which is consistent with the findings in Orphanides (2005). The focus on output growth rather than the output gap turns out to be crucial for the model to generate a fall in the Federal Funds rate. In response to a news shock, the output gap in the model increases whereas output growth falls.\(^\text{13}\) Hence, if monetary policy responded strongly to the output gap, this would reduce (or even reverse) the already insufficient drop on impact.

\(^{11}\)For $\sigma_u = 0$, depreciation increases linearly with utilization. In the absence of investment adjustment cost (i.e. $S'' = 0$), $\sigma_u = 0$ is inconsistent with the stationarity assumption for interest rates. See the appendix for details. We therefore impose a lower bound of $\sigma_u = 0.001$ on the estimation.

\(^{12}\)The estimate of $\rho = 0$ is not necessarily inconsistent with the literature. For example, Rudebusch (2006) argues that the persistence of the Federal Funds rate in response to a monetary policy shock is better explained by persistence in the exogenous shock process than persistence in the policy rule itself.

\(^{13}\)As described above, the output gap is defined as the difference between actual output and potential output in the absence of nominal price and wage rigidities. In response to a news shock, prices drop abruptly, which means that firms’ average markups decrease. Hence, actual output drops less than potential output (for which markups are constant by definition) and the output gap jumps up. Simulations with more basic policy rules that feature only the output gap show that the fit of the model under this specification falls apart almost completely, with the Federal Funds rate and the slope hardly responding to the news shock.
in the Federal Funds rate. A strong response to output growth, by contrast, reinforces the accommodative stance of the Fed thus bringing the model closer to the observed term structure dynamics.\textsuperscript{14}

Finally, consider the estimates of the risk loadings on expected inflation $\tilde{\lambda}_{1,\pi}^{\text{news}}$ and the expected change in marginal utility from consumption $\tilde{\lambda}_{1,\Delta u_{c}}^{\text{news}}$. Both coefficients are negative and highly significant. This provides another piece of evidence against a pure form of the Expectations Hypothesis and suggests that a linearized DSGE model on its own is incapable of generating sufficiently large term structure movements.

The estimates for the marginal cost coefficient of $\kappa_{p} = 1$ and the degree of wage rigidity $\xi_{w} = 0.99$ exceed other limited- and full-information estimates for the two coefficients in the literature, which typically reports values for $\kappa_{p}$ around 0.025 and for $\zeta_{w}$ around 0.75.\textsuperscript{15} To assess the model's performance conditional on more realistic price and wage adjustment processes, we fix $\kappa_{p} = 0.05$ and $\zeta_{w} = 0.85$. Both of these values are upper bounds found in the literature. The other parameters of the model are reestimated and are reported in the column labeled 'constrained estimates' of Table 2. The NKPC is still estimated to be completely forward-looking (i.e. $\omega_{p} = 0$) and the degree of wage indexation to past inflation goes to its upper bound of $\omega_{w} = 1$. Also, the model still favors highly variable capital utilization and no adjustment cost to investment. The intuition for these estimates is as above: for inflation to fall on impact of the shock, the NKPC needs to be driven by current and future marginal cost. Marginal cost, in turn, needs to be insensitive to the initial upward pressure coming from the negative income effect on labor supply. For the interest rate rule, there are relatively important changes, indicating that there is an important interplay between price and wage rigidity and monetary policy. In particular, the interest rate rule now exhibits substantial persistence (i.e. $\rho = 0.71$) and the response to output growth is about four times smaller.\textsuperscript{16}

Figure 5 plots the IRFs for the reestimated model. The model still generates an initial jump in consumption but can no longer match the subsequent increase to the new balanced growth level. By contrast, the model now implies a marked drop in output and investment on impact of the shock. The model also generates inflation dynamics that are reasonably

\textsuperscript{14}Barsky and Sims (2009) argue in favor of a similar monetary policy rule that does not respond to the output gap. However, their argument is somewhat different, based on their empirical result that real short-term rates in response to a TFP news shock are positive. As we pointed out above, however, real short-term rates are negative after a TFP news shock if inflation is measured by the more inclusive GDP deflator rather than the CPI deflator.

\textsuperscript{15}For example, Smets and Wouters' (2007) full-information estimation yields $\kappa_{p} = 0.021$ and $\xi_{w} = 0.73$.

\textsuperscript{16}In this 'constrained estimation', the issue of weak identification mentioned above manifests itself in an interest rate rule that has a tendency for $\theta_{\pi}$ and $\theta_{\Delta y}$ to wander off to unreasonably large values. We therefore fix $\theta_{\pi} = 3$. None of the results change when we fix $\theta_{\pi}$ to other values above 3.
close to the VAR counterpart. On the term structure side, however, the model now performs markedly worse. While the short rate still drops on impact, this drop is only about a third of what we see in the VAR. As a result, the spread implied by the Expectations Hypothesis barely increases and remains well below the VAR response. The model thus requires an even larger term premium on impact of the shock to generate a sufficiently large increase in the observed spread, which means that there is an even larger overshooting of the long rate on impact.

In sum, once we restrict the model to realistic parameter values for wage and price adjustment dynamics, the model cannot match the quantitative responses of term structure variables to a news shock. Modern New Keynesian DSGE models as proposed by Smets and Wouters (2007) thus fail to match the quantitative response of both macro and term structure variables to a TFP news shock. Since this class of DSGE models is commonly used for monetary policy analysis this failure represents an important challenge for modern macroeconomics.

5 Conclusion

Our results provide an important benchmark to evaluate theories of the term structure and, more generally, DSGE models. We show that a medium-scale DSGE model along the lines of Smets and Wouters (2007) falls well short of matching the term structure response to a TFP news shock that we see in the data. This failure of generating realistic term structure dynamics is problematic for two reasons. First, asset prices (of longer-term securities in particular) are an important determinant of consumption and investment decisions. If a DSGE model cannot simultaneously match both macroeconomic and asset price dynamics, then this suggests a serious empirical shortcoming of theory. Second, medium-scale DSGE models are increasingly used for monetary policy analysis. If these models fail to generate reasonable term structure dynamics, then it seems difficult to trust them for the evaluation of how monetary policy transmits into the economy. A fruitful path for future research is to search for a mechanism to augment DSGE models that provide a more detailed and realistic mechanism of technology diffusion and the frictions involved in factor allocation and reallocation. By the same token, we need to better understand the monetary policy reaction to TFP news shocks. Without these two issues resolved, we cannot hope to generate realistic term structure movements, even for the expectational part and let alone the part due to

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17 It is possible to find parameter combinations that allow the model to match the responses of the term structure data almost perfectly. The problem is that for these parameter combinations the model's performance for the macroeconomic variables deteriorates significantly.
term premium variations.
References


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<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Smets-Wouters estimates</th>
<th>Unconstrained estimates</th>
<th>Constrained estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_{\mu_A}$</td>
<td>Persistence of TFP growth</td>
<td>0.837 (0.037)</td>
<td>0.837 (0.037)</td>
<td>0.837 (0.037)</td>
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<tr>
<td>$\sigma_{\text{news}}$</td>
<td>Standard deviation of news shock</td>
<td>0.061 (0.018)</td>
<td>0.061 (0.018)</td>
<td>0.061 (0.018)</td>
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<td>$\kappa_p$</td>
<td>Marginal cost slope of NKPC</td>
<td>0.021</td>
<td>1.000 (n.a.)</td>
<td>0.05 (n.a.)</td>
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<td>$\omega_p$</td>
<td>Degree of price indexation</td>
<td>0.228</td>
<td>0 (n.a.)</td>
<td>0 (n.a.)</td>
</tr>
<tr>
<td>$\xi_w$</td>
<td>Probability of wage non-adjustment</td>
<td>0.27</td>
<td>0.992 (0.006)</td>
<td>0.85 (n.a.)</td>
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<tr>
<td>$\omega_w$</td>
<td>Degree of wage reoptimization</td>
<td>0.59</td>
<td>0.695 (0.025)</td>
<td>1 (n.a.)</td>
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<td>$b$</td>
<td>Habit persistence</td>
<td>0.71</td>
<td>0.880 (0.079)</td>
<td>0.461 (0.206)</td>
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<tr>
<td>$\sigma_u$</td>
<td>Capital utilization parameter</td>
<td>1.174</td>
<td>0.006 (0.005)</td>
<td>0.0165 (0.030)</td>
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<td>$S^*$</td>
<td>Investment adjustment cost</td>
<td>5.48</td>
<td>0 (n.a.)</td>
<td>0 (n.a.)</td>
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<tr>
<td>$\rho_R$</td>
<td>Persistence of interest rate rule</td>
<td>0.81</td>
<td>0 (n.a.)</td>
<td>0.710 (0.151)</td>
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<tr>
<td>$\theta_{\pi}$</td>
<td>Inflation response</td>
<td>2.03</td>
<td>2.920 (0.504)</td>
<td>3.000 (n.a.)</td>
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<tr>
<td>$\theta_{y_{gap}}$</td>
<td>Output gap response</td>
<td>0.08</td>
<td>-0.015 (0.002)</td>
<td>0.165 (0.226)</td>
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<tr>
<td>$\theta_{\Delta y}$</td>
<td>Output growth response</td>
<td>0.22</td>
<td>2.130 (0.012)</td>
<td>0.553 (0.265)</td>
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<td>$\lambda^\text{news}_{1,\pi}$</td>
<td>Risk loading on expected inflation</td>
<td>0</td>
<td>-6.999 (1.005)</td>
<td>-11.165 (3.390)</td>
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<tr>
<td>$\lambda^\text{news}_{1,\Delta u_c}$</td>
<td>Risk loading on expected change in marginal utility from consumption</td>
<td>0</td>
<td>-15.032 (1.771)</td>
<td>-15.594 (0.322)</td>
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Figure 4